KENNESAW STATE UNIVERSITY
COLLEGE OF SCIENCE AND MATHEMATICS
Department of Mathematics

THE 2025–2026 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. In a movie theater line, x people are behind Keisha, who is y people in front of Sam. If there are z people in front of Sam, how many people are in the line?

(A) $z - x + y + 2$ (B) $z + x - y$ (C) $z - x + y - 1$ (D) $z + x - y + 1$ (E) $z - x + y$

2. The number of seconds in 6 weeks equals $n!$. What is the value of n?

(A) 10 (B) 12 (C) 14 (D) 16 (E) 18

3. The numerator and denominator of a fraction are positive integers that differ by 8. The value of the fraction is strictly between $\frac{4}{7}$ and $\frac{3}{5}$. Compute the sum of the numerator and denominator of this fraction.

(A) 18 (B) 21 (C) 24 (D) 27 (E) 30

4. The average of nine non-negative numbers is 10. What is the greatest possible value for their median?

(A) 17 (B) 17.5 (C) 18 (D) 18.5 (E) 19

5. If I stand still on an escalator, I can move from one floor of a building to the next in 20 seconds. If I walk at my standard uniform pace on a parallel set of stairs, the trip takes 15 seconds. How many seconds will the trip take if I walk on the escalator at my standard uniform pace?

(A) $8\frac{1}{2}$ (B) $8\frac{2}{3}$ (C) $8\frac{3}{5}$ (D) $8\frac{4}{7}$ (E) $8\frac{5}{9}$

6. The function f has the property $f(x) + f(x - 1) = 1$ for all real values of x. If $f(3) = 3$, compute $f(2026)$.

(A) -2 (B) -1 (C) 1 (D) 2 (E) 3

7. Let K = the number of digits in 2^{2025} and N = the number of digits in 5^{2025}. Compute $K + N$

(A) 2024 (B) 2025 (C) 2026 (D) 4050 (E) None of these

8. In the xy-plane, line ℓ passes through the point Q(4, 9). If the x-intercept of ℓ is a positive prime number and the y-intercept is a positive integer, compute the sum of the x-intercepts of all such lines ℓ.

(A) 5 (B) 12 (C) 19 (D) 25 (E) 30

9. If i is the imaginary unit, and a and b are real numbers, then the conjugate of the complex number $z = a + bi$ is represented by $\bar{z} = a - bi$. If $(z + \bar{z})z = 2 + 4i$, compute $a^2 + b^2$.

(A) 5 (B) 10 (C) 13 (D) 17 (E) 20

10. In Dr. Garner's math class, all the students were either juniors or seniors. Two-thirds of the juniors and one-half of the seniors passed the last math test. If there are three-fourths as many juniors as seniors in the class, what fraction of the entire class passed the last math test?

(A) $\frac{4}{7}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{7}{9}$ (E) None of these

11. If $x + \frac{1}{y} = 4$, $y + \frac{1}{z} = 1$, and $z + \frac{1}{x} = \frac{7}{3}$, compute the value of xyz.

(A) 4 (B) 2 (C) 1 (D) $\frac{1}{3}$ (E) $\frac{4}{3}$

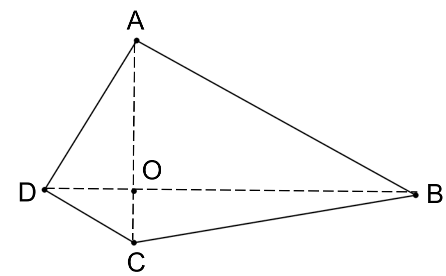
12. Two different numbers are taken from the set $\{0,1,2,3,4,5,6,7,8,9,10\}$. Determine the probability that their sum and the absolute value of their difference are both multiples of 4.

(A) $\frac{1}{11}$ (B) $\frac{6}{55}$ (C) $\frac{8}{55}$ (D) $\frac{9}{55}$ (E) $\frac{2}{11}$

13. Compute the sum $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 2023^2 - 2024^2$

(A) −2047276 (B) −2049300 (C) −4072324 (D) −4074342 (E) −4076361

14. In the figure, ABCD is a quadrilateral with sides of integral length, all unequal. Diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular. Given that AB = 8 and BC = 7, compute the perimeter of the quadrilateral.



(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

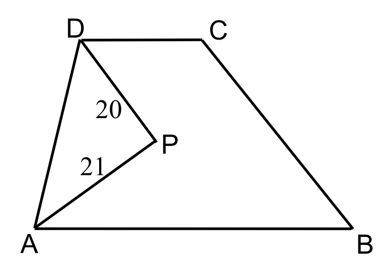
15. Three integers form a geometric sequence. If the second is increased by 8 they will form an arithmetic sequence. However, if after that, we increase the last number by 64, the sequence becomes geometric again. Compute the sum of these three integers.

(A) 48 (B) 52 (C) 54 (D) 56 (E) 60

16. A group of 200 people, some adults and some children, attend a banquet. Because of a special promotion, each adult pays twice as much as each child, and each child pays the same amount. The total bill is exactly $1,125. If each person pays a whole number of dollars, what is the largest possible number of adults in the group?

(A) 100 (B) 125 (C) 150 (D) 175 (E) 200

17. In trapezoid ABCD, with bases AB and CD, the bisector of angle D and the bisector of angle A meet at interior point P. If DP = 20, and AP = 21, compute the length of the altitude of the trapezoid.



(A) $\frac{510}{29}$ (B) $\frac{620}{29}$ (C) $\frac{730}{29}$ (D) $\frac{840}{29}$ (E) None of these

18. Let $A_n = 1 + 3 + 5 + \ldots + (2n - 1)$, where n is a positive integer and $B_n = \log A_1 + \log A_2 + \ldots + \log A_n$. If $B_6 + B_7 = B_x$ for some integer x, compute x.

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

19. If $\tan(11x°) = \tan(12°)$ and $\tan(19x°) = \tan(168°)$, compute $\tan(5x°)$.

(A) $-\sqrt{3}$ (B) $\frac{-\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{3}$ (E) 1

20. A three-digit positive integer *abc* is twice as big in base 10 as the three-digit positive integer *abc* in base 7. How many such three-digit positive integers are there? (Note: 0*bc* is not a three-digit number.)

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

21. The roots of the equation $x^3 + ax^2 + bx + c = 0$ are each two less than the roots of the equation $x^3 + x + 1 = 0$. Compute $a + b + c$.

(A) 27 (B) 28 (C) 29 (D) 30 (E) 31

22. On a standard die, the sum of the numbers on opposite faces is 7. When seven standard dice are rolled, the probability that a sum of 10 occurs is $\frac{7}{23328}$. What other sum occurs with the exact same probability?

(A) 36 (B) 37 (C) 38 (D) 39 (E) 40

23. Find the sum of all integers x such that $x^2 + 17x + 42$ is the square of a non-zero integer.

(A) –34 (B) –17 (C) 17 (D) 34 (E) 42

24. Triangle ABC with AB = 7, BC = 19, and AC = 24, is inscribed in a circle. Chord $\overline{BD}$ intersects $\overline{AC}$ at point E. If BE = 5, compute the smallest possible length for $\overline{ED}$.

(A) 12.6 (B) 12.8 (C) 13.4 (D) 13.6 (E) 14.2

25. Let $K = 1375^n - 1653^n - 1746^n + 2024^n$. K is divisible by exactly two four-digit numbers for all $n \geq 0$. Compute the sum of these two numbers.

(A) 6798 (B) 8415 (C) 9313 (D) 10307 (E) 11231

Solutions

1. D Since Keisha is y places in front of Sam, there are $x - y$ people behind Sam, excluding Sam. Then, since there are z people in front of Sam, again excluding Sam, the total number of people in the line is:

number behind Sam + number in front of Sam + Sam = $x - y + z + 1$

2. A The number of seconds in 6 weeks is $6 \cdot 7 \cdot 24 \cdot 60 \cdot 60 = 6 \cdot 7 \cdot 24 \cdot 30 \cdot 5!$ $= 7! \cdot 24 \cdot 30 = 7! \cdot 8 \cdot 3 \cdot 3 \cdot 10 = 10!$. Thus, $n = 10$.

3. E Represent the fraction by $\frac{a-8}{a}$. Then

$\frac{a-8}{a} > \frac{4}{7} \Rightarrow a > 18\frac{2}{3}$ and $\frac{a-8}{a} < \frac{3}{5} \Rightarrow a < 20$.

Since a is an integer, $a = 19$. The fraction is $\frac{11}{19}$, and the desired sum is 30.

4. C Since the average of the nine numbers is 10, there sum is 90. If the median is m, then the five highest numbers are all at least m, so the sum of all the numbers is at least $5m$. Thus $90 \geq 5m \Rightarrow m \leq 18$. Conversely, we can achieve $m = 18$ by taking four 0's and five 18's. Therefore, the maximum median is 18.

5. D Set up a table.

	Rate	Time	Distance
Escalator	E	20	$20E$
Walk	W	15	$15W$
Both	$E + W$	t	

$$t = \frac{distance}{rate} = \frac{15W}{E+W} = \frac{15W}{\frac{3}{4}W+W} = \frac{15}{\frac{7}{4}} = \frac{60}{7} = 8\frac{4}{7}.$$

6. A We are given $f(x) + f(x-1) = 1$. Then $f(x) = 1 - f(x-1)$ and $f(x+1) = 1 - f(x)$. Thus, $f(x+1) = 1 - [1 - f(x-1)] = f(x-1)$.

Establishing a pattern:

Given $f(x) = 1 - f(x-1)$ and $f(3) = 3$

Then $f(4) = 1 - f(3) = 1 - 3 = -2$

$f(5) = 1 - f(4) = 1 - (-2) = 3$

$f(6) = 1 - f(5) = 1 - 3 = -2$

Thus, when x is odd, $f(x) = 3$ and when x is even, $f(x) = -2$. Hence $f(2026) = -2$

.

7. C 10^{K-1} is the smallest integer with K digits, and 10^K is the smallest with $K + 1$ digits. Since neither 2^{2025} nor 5^{2025} is exactly a power of 10, we know that

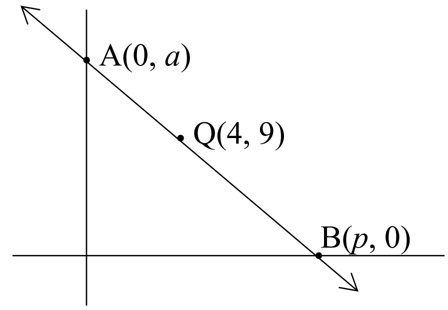
$$10^{K-1} < 2^{2025} < 10^K \text{ and } 10^{N-1} < 5^{2025} < 10^N.$$

If we multiply these inequalities (all sides are positive), we get

$$10^{K+N-2} < (2^{2025})(5^{2025}) < 10^{K+N}.$$

The product in the middle is 10^{2025}, therefore $K + N - 2 < 2025 < K + N$, from which $K + N = 2026$.

8. D Let the y-intercept be A(0, a) and the x-intercept B(p, 0).



The the equation of line ℓ is $y = \frac{9}{4-p}x + a$. Substituting (4, 9), clearing fractions and simplifying, $9p = ap - 4a \Rightarrow a = \frac{9p}{p-4}$.

The only primes, p, that will give a positive integer value of a are 5, 7, and 13. The desired sum is 25.

9. A $(z + \bar{z})z = (a + bi + a - bi)(a + bi) = 2(a^2 + abi) = 2 + 4i = 2(1 + 2i)$. Therefore, $a^2 = 1$ and $ab = 2 \Rightarrow a = \pm 1, b = \pm 2$ so that $a^2 + b^2 = 1 + 4 = 5$.

10. A Let n be the number of seniors in the class. Then there are $\frac{3}{4}n$ juniors in the class.

Thus, the number of students who passed the last test was $\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)n + \frac{1}{2}n = n$;

and there are $n + \frac{3}{4}n$ students in the class. Therefore, the fraction of students that passed the last test is $\frac{n}{\frac{7}{4}n} = \frac{4}{7}$.

11. C Noting that $x = 4 - \frac{1}{y}$, $y = 1 - \frac{1}{z}$, and $z = \frac{7}{3} - \frac{1}{x}$

Multiply the equations:

$\frac{28}{3} = 4 \cdot 1 \cdot \frac{7}{3} = \left(x + \frac{1}{y}\right)\left(y + \frac{1}{z}\right)\left(z + \frac{1}{x}\right) = xyz + x + y + z + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{xyz}$

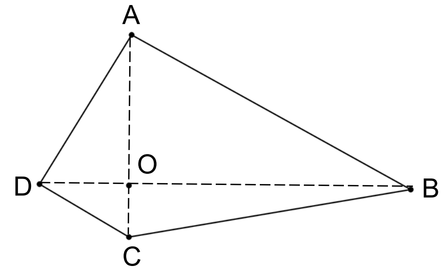
$= xyz + \frac{22}{3} + \frac{1}{xyz}$. Thus, $xyz + \frac{1}{xyz} = 2 \Rightarrow (xyz - 1)^2 = 0 \Rightarrow xyz = 1$.

12. B Let the two numbers be a and b, with $a > b$. Since $a + b = 4x$ and $a - b = 4y$, where x and y are integers. Then, $a = 2(x + y)$ and $b = 2(x - y)$. Thus, a and b are two even numbers that differ by a multiple of 4. There are 6 such pairs, (0,4), (0,8), (4,8), (2,6), (2,10), and (6,10) and ${}_{11}C_2 = 55$ total pairs. Therefore, the desired probability is $\frac{6}{55}$.

13. B Grouping the terms in pairs from left to right and factoring

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + 2023^2 - 2024^2 =$$
$$(1 - 2)(1 + 2) + (3 - 4)(3 + 4) + \ldots + (2023 - 2024)(2023 + 2024) =$$
$$(-1)(1 + 2 + 3 + 4 + \ldots + 2024) = -\frac{(2024)(2025)}{2} = -2049300.$$

14. A Let O be the point of intersection of the two diagonals. Then Using the Pythagorean Theorem on each of the four right triangles,



$(AB)^2 = (OA)^2 + (OB)^2$
$(BC)^2 = (OB)^2 + (OC)^2$
$(CD)^2 = (OC)^2 + (OD)^2$
$(DA)^2 = (OD)^2 + (OA)^2$.

Thus, $(AB)^2 + (CD)^2 = (OA)^2 + (OB)^2 + (OC)^2 + (OD)^2 = (BC)^2 + (DA)^2$ Substituting AB = 8 and BC = 7, we obtain $(DA)^2 - (CD)^2 = 15$. Then $(DA - CD)(DA + CD) = 15$. Since all sides have integral length, there are two possibilities: DA + CD = 5 and DA – CD = 3 or DA + CD = 15 and DA – CD = 1. The second case gives DA = 8 which is impossible since we are given that all side lengths are unequal. The first case gives DA = 4 and CD = 1, and the perimiter of ABCD is 20.

15. B Let the terms of the original geometric sequence be a, ar, ar^2. Then by the given
(i) $a, ar + 8, ar^2$ is an arithmetic sequence
(ii) $a, ar + 8, ar^2 + 64$ is a geometric sequence

From (ii) $a(ar^2 + 64) = (ar + 8)^2$, which can be rewritten $a = \frac{4}{4-r}$.

From (i) $a + ar^2 = 2(ar + 8)$. Substituting $a = \frac{4}{4-r}$ and simplifying, we get

$r^2 + 2r - 15 = 0$. Hence $r = -5$ and $r = 3$. The corresponding values of a are $\frac{4}{9}$ and 4.

Since the three given numbers are integers, $a = 4, r = 3$, and the original sequence is 4, 12, 36 with a sum of 52.

16. D Let A = the number of adults, C = the number of children, and D = the number of dollars paid by each child. Then

$A + C = 200$ and $2D\cdot A + D\cdot C = 1125$

Substituting $C = 200 - A$ into the second equation, $2D\cdot A + D(200 - A) = 1125$ or

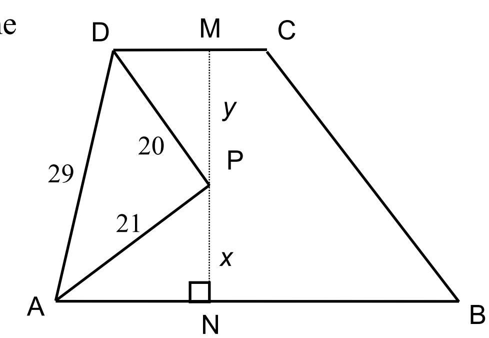
$D\cdot A + 200D = D(A + 200) = 1125 = (5^3)(3^2)$.

Therefore, the possible combinations for D and $A + 200$ (in either order) are

1, 1125; 5, 225; 25, 45; 125, 9; 3, 375; 15, 75.

Since $A < 200$, $200 < A + 200 < 400$, so the only possibilities for $A + 200$ are 225 and 375. Therefore, the largest possible number of adults in the group is 175.

17. D Because ∠CDA and ∠DAB are supplementary, the sum of the measures of ∠PDA and ∠PAD = $\frac{1}{2}(180) = 90°$. Therefore, ΔDPA is a right triangle. Using the Pythagorean Theorem, AD = 29. Construct the altitude of the trapezoid through P, intersecting $\overline{AB}$ at N and $\overline{CD}$ at M.



Because ∠DAP ≅ ∠PAN, ΔAPN ~ ΔADP. Let PN = x.
Because ∠ADP ≅ ∠PDM, ΔPDM ~ ΔADP. Let PM = y.
Therefore,

$\frac{x}{20} = \frac{21}{29}$ and $\frac{y}{21} = \frac{20}{29}$. Solving, $x = y = \frac{420}{29}$ and the length of $\overline{MN}$ is $x + y = \frac{840}{29}$.

18. B Note that $A_n = n^2$, and $B_n = \log(A_1 A_2 \cdots A_n) = \log(1^2 2^2 \cdots n^2) = 2\log(n!)$. Then, $B_6 + B_7 = B_x$ implies $2\log(6!) + 2\log(7!) = 2\log(6!\cdot 7!) = 2\log(x!)$, so $x! = 6!\cdot 7! = 10!$. Thus, $x = 10$.

19. A $\tan(a) = \tan(b)$ if and only if $a = b + 180k$ for some integer k. Therefore, $11x = 12 + 180k$ and $19x = 168 + 180m$, for some k and m.
This implies $\tan 5x = \tan(2\cdot 19x - 3\cdot 11x) = \tan[2\cdot 168 - 3\cdot 12 + (2m - 3k)180] = \tan(2\cdot 168 - 3\cdot 12) = \tan(300) = -\sqrt{3}$.
Note: $x = 132 + 180k$

20. D Let the 3-digit number be abc. Then (in base 10) we have

$$100a + 10b + c = 2(49a + 7b + c) = 98a + 14b + 2c \;\Rightarrow\; a = 2b + \frac{c}{2}.$$

Now, a, b, and c must be digits in base 7 (0 to 6), and $a \neq 0$ because it is the leading digit, so all the possible cases are

$$c = 0 \;\Rightarrow\; a = 2b \;\Rightarrow\; abc = 210, 420, 630$$
$$c = 2 \;\Rightarrow\; a = 2b + 1 \;\Rightarrow\; abc = 102, 312, 522$$
$$c = 4 \;\Rightarrow\; a = 2b + 2 \;\Rightarrow\; abc = 204, 414, 624$$
$$c = 6 \;\Rightarrow\; a = 2b + 3 \;\Rightarrow\; abc = 306, 516$$

Of these 11 solutions, 102 is not acceptable since 102 in base 7 has only two digits when written in base 10. Therefore, the total number of solutions is 10.

21. D **Method I:**
In any equation of the form $x^3 + ax^2 + bx + c = 0$ with roots p, q, and r,

(i) $p + q + r = -a$, (ii) $pq + pr + qr = b$, and (iii) $pqr = -c$

Let the roots of $x^3 + x + 1 = 0$ be p, q, r. We know that $p + q + r = 0$, $pq + pr + qr = 1$ and $pqr = -1$.

The roots of the equation $x^3 + ax^2 + bx + c = 0$ are $(p - 2)$, $(q - 2)$, and $(r - 2)$.

Now, $a = -[(p - 2) + (q - 2) + (r - 2)] = -[p + q + r - 6] = 6$. Also,
$b = (p - 2)(q - 2) + (p - 2)(r - 2) + (q - 2)(r - 2) = (pq + pr + qr) - 4(p + q + r) + 12 = 13$.
Also, $c = -[(p - 2)(q - 2)(r - 2)] = -[pqr - 2(pq + pr + qr) + 4(p + q + r) - 8] = -[-1 - 2 - 8] = 11$.
Therefore, the desired equation is $x^3 + 6x^2 + 13x + 11 = 0$ and $a + b + c = 30$.

Method II:
Let p be a root to x^3 + x + 1 = 0. Then p^3 + p + 1 = (p - 2 + 2)^3 + (p - 2 + 2) + 1= 0. This means that p-2 satisfies the equation (x + 2)^3 + (x + 2) + 1 = 0. Expanding things out, we have x^3 + 6x^2 + 13 x + 11 = 0 and a + b + c = 6 + 13 + 11 = 30.

22. D Since the sum of the numbers on opposite faces is 7, the numbers on opposite faces can be represented generally by x and $7 - x$, where x is an integer and $0 < x < 7$. Therefore, when seven dice are rolled, a sum of S on the top of the dice corresponds to a sum of $49 - S$ on the bottom of the dice. If $a + b + c + d + e + f + g = S$, then $(7 - a) + (7 - b) + (7 - c) + (7 - d) + (7 - e) + (7 - f) + (7 - g) = 49 - S$. We therefore conclude that rolling a sum of S is as

likely as rolling a sum of 49 – S. We may then conclude that sums of 10 and 39 are equally likely.

23. B First, notice that $x^2 + 17x + 42 = (x + 14)(x + 3)$. Then $x = -3$ and $x = -14$ make the value of the polynomial zero, which is a perfect square, but we are looking for a non-zero answer. Now, we complete the square to find

$x^2 + 17x + 42 = x^2 + 17x + \frac{289}{4} - \frac{121}{4} = \left(x + \frac{17}{2}\right)^2 - \frac{121}{4}$. Since this must be a square,

let $\left(x + \frac{17}{2}\right)^2 - \frac{121}{4} = m^2 \Rightarrow \left(x + \frac{17}{2}\right)^2 - m^2 = \left(x + \frac{17}{2} - m\right)\left(x + \frac{17}{2} + m\right) = \frac{121}{4}$.

Multiplying both sides by 4, $(2x + 17 - 2m)(2x + 17 + 2m) = 121$.

There are now four possibilities:

(1) $(2x + 17 - 2m) = 11$ and $(2x + 17 + 2m) = 11$

(2) $(2x + 17 - 2m) = -11$ and $(2x + 17 + 2m) = -11$

(3) $(2x + 17 - 2m) = 1$ and $(2x + 17 + 2m) = 121$

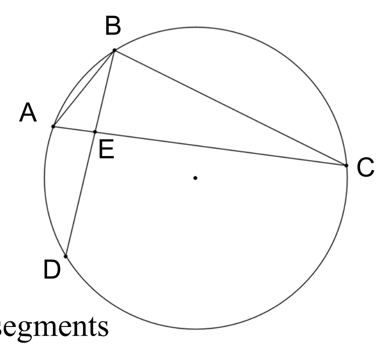
(4) $(2x + 17 - 2m) = -1$ and $(2x + 17 + 2m) = -121$

From (1) we get $x = -3$ and from (2) we get $x = -14$. These give the polynomial a value of zero, which we have already discussed.

From (3) we get $x = 22$ and the value of the polynomial is $900 = 30^2$

From (4) we get $x = -39$ and the value of the polynomial is again $900 = 30^2$.

Therefore, the desired sum is $22 + (-39) = -17$.



24. A Using the law of cosines on ΔABC,

$$19^2 = 7^2 + 24^2 - 2(7)(24)\cos A \Rightarrow \cos A = \frac{264}{336} = \frac{11}{14}$$

Letting $AE = x$, and using the law of cosines on ΔABE

$$5^2 = 7^2 + x^2 - 2(7)(x)\left(\frac{11}{14}\right) \Rightarrow x^2 - 11x + 24 = 0 \Rightarrow x = 3, 8$$

When two chords of a circle intersect, the product of the lengths of the segments on one chord is equal to the product of the lengths of the segments of the other.

If $AE = 3$, then $(BE)(ED) = (AE)(EC) \Rightarrow (5)(ED) = (3)(21) \Rightarrow ED = 12.6$.

If $AE = 8$, then $(BE)(ED) = (AE)(EC) \Rightarrow (5)(ED) = (8)(16) \Rightarrow ED = 25.6$

Therefore, the smallest possible value for $ED = 12.6$.

25. C We are given $K = 1375^n - 1653^n - 1746^n + 2024^n$. Since $x - y$ is a factor of $x^n - y^n$ for all $n \geq 0$,

$2024^n - 1746^n$ is divisible by $2024 - 1746 = 278$ and

$1375^n - 1653^n$ is divisible by $1375 - 1653 = -278$ (and thus divisible by 278).

Similarly,

$2024^n - 1653^n$ is divisible by $2024 - 1653 = 371$ and

$1746^n - 1375^n$ is divisible by $1746 - 1375 = 371$.

Since 278 and 371 are relatively prime, the given expression must be divisible by $(278)(371) = 2 \cdot 7 \cdot 53 \cdot 139$. Therefore, the 2 four-digit numbers we are looking for are $2 \cdot 7 \cdot 139 = 1946$ and $53 \cdot 139 = 7367$, with a sum of 9313.