Informativeness of News, Signal Quality and Media Capture

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Abstract

We present a model of media bias with a single media outlet and consider two alternative environments: indirect government control (capture) and no government interference (freedom of press). The government wants to mobilize citizens to invest in a project that may not necessarily align with their best interest. In this context, we present a number of counter-intuitive results. We show that, under freedom of press, the media firm's report is unbiased (most informative) when citizens are less reliant on news and and biased in the direction of their prior (least informative) when they are more reliant. Finally, we show that the government captures the media outlet in order to maximize the expected number of investors, however, it does so by making the news completely uninformative.

KEY WORDS: *Informativeness; Freedom of Press; Media Capture; Signal Quality.*
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1 Introduction

Over the last few decades, perhaps as an aftermath of increasing economic and political polarization or from increasing interference of the political elites, trust in the media as an unbiased source of information has been waning (Ladd (2012)). Since citizens depend on the news to deliver information on matters of importance, accuracy of information on issues they report is of great concern. However, since citizen's trust in media is declining, we must ask what factors hinder the media's ability to report news accurately. Consequently, to shed more light on the interplay of factors that determine the information content of news, we consider a model with the following agents: a government, a private media firm and a continuum of citizens of size one.

We assume that the agents have a prior about the binary but unobserved state of the economy, namely, good (G) and bad (B). The government wants to mobilize citizens to take some action that serves its own interest but may or may not align with the interest of citizens. This may represent situations where the government wants citizens to support some policy initiative like repealing government mandated healthcare, to form or end some treaty like Brexit, to support a war, or vote to keep the incumbent in power, etc. In the context of this model we will operationalize the notion of taking some action as 'investment'. Citizens incur a cost to invest and gain from investing only if the state of the economy is G. The media firm receives a noisy signal about the actual state and reports the likely state. The government offers a bribe to the media firm to report the state that favors investment. The media firm can either accept or reject the bribe. If it rejects the bribe then it reports its signal in order to maximize profit (which we call *freedom of press*). The firm's profit is assumed to depend on the size of its subscribers and the reward it may receive from predicting the actual state accurately. If it accepts the bribe its report furthers the interest of the government (which we call *capture*). With this setup, we present a number of counter-intuitive results as noted below.

First, we show that the media firm, under *freedom of press*, always reports its signal truthfully when it receives the signal that the state of the economy is good but may choose to garble the signal when it receives the signal that the state of the economy is bad. Further, we show that the media firm's report (under *freedom of press*) is unbiased (most informative) when the citizens are less reliant on the news i.e., when the cost of investment is below a certain threshold and biased (the bad signal is garbled with some probability), i.e., least informative when citizens are more reliant on the news, which occurs when the cost of investment is above a certain threshold. The intuition behind the result is as follows; When the cost of investment is low, citizens have little to lose when the actual state turns out to be bad. In other words, when investment is not costly, then adhering to the news report is not vital because the loss is small (in that case we find that the difference between the minimum return that justifies investment with and without the news is smaller). Hence, citizens are less tolerant of the bias. As a consequence, the media firm becomes most informative to avoid losing subscribers. Now consider the case where investments are costly. Since the citizens have more to lose, they are more tolerant of bias because the news media, even with some bias, provides some information of value about the state of the economy.

In contrast, the government captures the media in order to mobilize the maximum number of citizens to invest in the project. However, it achieves this by making the news completely uninformative. In that case citizens no longer subscribe to the news but make investment decisions based on the prior (which happens because the signal quality is below a certain threshold). Stated alternatively, the government uses complete misinformation to discourage citizens from following the news in order to maximize the number of investors. The intuition behind the result is as follows: the government's payoff depends on the expected number of investors which is a product of the likelihood of receiving the good report and the number of citizens who invest. In the absence of news, a fraction of the population always invest in the project based on the prior. As it turns out, when the quality of the signal is below a certain threshold, the expected number of investors mobilized by the news is smaller than the expected number of investors in the absence of news (i.e.,when citizens invest based on the prior). Hence, the government increases the expected number of investors by making the news completely uninformative. Finally we also show that capture becomes more attractive to the government as signal quality of the media firm decreases or the dis-utility from reading news increases.

2 Related Literature

In the literature, on the causes and consequences of media bias, bias may exist either because of the news outlets' response to consumer preferences either exogenously (Mulainathan and Shleifer (2005), Bernhardt et al. (2008), Duggan and Martinelli (2011), Dyck et al. (2008) and Strömberg (2004)) or because it may serve as a tool to enhance reputation (Gentzkow and Shapiro (2006)). Bias may also arise as a result of inherent biases of news-makers (Gabszewicz et al. (2004), Baron (2006) and Puglisi (2011)) or because of the capture of journalists by lobbies to further their interest (Corneo (2006), Petrova (2008) and Petrova (2012)). It could also arise from the capture of news outlets by the government (Besley and Pratt (2006) and Gelbach and Sonin (2014)) or because of control exercised by dictators or autocracies (Debs (2010), Edmonds (2008), Lorentzen (2013), Egorov et al. (2009) and Qin et al. (2018)).

None of the papers mentioned above analyze the role of investment/mobilization costs and signal quality in determining information content of news with or without government interference. In this context, our model is closest to Roy (2019) in which he explores the role of investment cost and signal quality to compare media bias across capture and free press and underline conditions under which one may be smaller or greater than the other. In his model, the news is neither completely informative nor completely uninformative in any scenario. Our model is different in the following respects: we present a counter-intuitive result that the government makes the news completely uninformative to maximize mobilization which cannot occur, in equilibrium, in his model. Further, unlike his paper, we also show that the media firm (under free press) carries no bias when citizens stand to lose less from investment but biases its report when they stand to lose more. Finally, we show that capture is more likely when the cost of investment is lower or when the dis-utility from news is higher which is not analyzed in his model.

3 Model

There are two possible but unobservable states of the economy $S \in \{G, B\}$ and three types of agents in the economy: a government that wants to mobilize its citizens to invest in a project that may not necessarily align with their own interest. A continuum of citizens of unit mass with each citizen expecting a private return X on the project from a known distribution $U(0, 2m)$ when the state of the economy is good. The project is not expected to yield a return in the bad state. The cost of investing in the project is $c > 0$. As a consequence, it is not in the interest of citizens to invest in the project in the bad state. Finally, there's a media firm that receives imperfect signals about the state of the economy. The media firm reports its signals with or without government interference.

The government receives a monetary benefit of γ for each citizen who invests in the project. The state $S = G$ is expected to occur with probability θ. Since the state is unobserved, the government offers a bribe to the media firm to influence its report. The firm may choose to accept or reject the bribe. If it accepts the bribe then its report aligns with the objective of the government. If it rejects the bribe then it chooses to report its signal in a way that maximizes profits. Citizens are assumed to be Bayesian. They read the media firm's report and update their beliefs about the likely state and decide whether or not to invest. The timing of events, similar to Baron (2006) and Gelbach and Sonin (2014), is as follows:

a. The unobservable state $S \in \{G, B\}$ is realized.

b. The government offers a bribe T to influence the report released by the media firm.

c. The media firm decides whether or not to accept the bribe and makes it editorial policy (how to report its signal) public.

d. Citizens subscribe to the news.

e. The media firm receives the signal $s \in \{g, b\}$. The quality of the signal is common knowledge and is represented by: $q = \Pr[s = g|S = G] = \Pr[s = b|S = B]$ and $(1 - q) = Pr[s = g|S = B] = \Pr[s = b|S = G]$. Since the signal is noisy we must have $q < 1$. Further, we assume that $q > \frac{1}{2}$.[1] The media firm reports $r^i \in \{g, b\}$ after receiving the signal in accordance with its editorial policy.

f. Citizens read the report, update their priors and decide whether or not to invest.

g. The payoffs are realized.

In what follows we outline the payoff functions for the media firm and the government. Since it is possible for the media firm to be bribed, we identify whether or not the media firm is acting out of its own self interest by the following index $i \in \{P, Gov\}$ where $i = Gov$ stands for *capture* and $i = P$ represents the case when the media firm is *free*, i.e., *freedom of press.* The media firms payoff comes from advertising revenues, the reward for accurately predicting the actual state and the bribe offered by the government net of the cost of producing news. Let a be the revenue received per subscriber and $\tilde{S}_i$ be the number of subscribers. The payoff functions is given by:

$$\pi_i = a\tilde{S}_i + r(\sigma, \tilde{S}_i; q) + T_i - K \tag{1}$$

where $r(.)$ represents the reward from accurately predicting the actual state, which captures attributes like awards, prizes, prestige associated with accurate prediction. Further, we assume that $r(.)$ is a function of the distortion σ which represents the probability with which the media firm may choose to report a signal which is different from the one it actually receives, the size of the audience/subscribers, $\tilde{S}_i$, and the quality of the signal q. The cost of producing news is captured by K and is assumed to be constant in the long run (see Strömberg (2004), Rosse and Dertouzos (1978), Thompson (1989)). Finally T_i represents the bribe that the media firm receives from the government in the event of capture. We assume, for the sake of simplicity, that the subscription price for reading or following the news is zero as the case maybe for TV or for most online news portals. Our analysis, however, carries over even with positive subscription prices. We lay out the sketch of the proofs with subscription price in Appendix B.

[1]This implies that the media firm's signal is trustworthy with probability greater than half. Citizens do not subscribe to the news if $q < \frac{1}{2}$.

Now let us consider the payoff of the government. Let I_j^i denotes the number of citizens who invest following a report $r^i \in \{g, b\}$ and $i \in \{Gov, P\}$. As mentioned before, the government's payoff depends entirely on the number of investors less of the bribe it pays to the media firm. Therefore, the payoff the government can be represented by:

$$V_{Gov} = \gamma \sum_{j \in \{g,b\}} \Pr\big[r^i = j\big] I_j^i - T_i \tag{2}$$

Clearly, $T_P = 0$ since under *freedom of press* the government's bribe is rejected. We proceed backwards and use Perfect Bayesian Nash Equilibrium (PBNE) to solve the game.

4 Subscription Choice

Let $d \geq 0$ be the dis-utility a citizen incurs from reading or following the news. Since the citizens are Bayesian, they infer the likelihood of the state G after receiving news $r^i = j$ for $j \in \{g, b\}$ using Bayes' theorem. Therefore, the posterior probability of G given $r^i = g$ is given by:

$$\begin{aligned} \Pr\big[S = G | r^i = g\big] &= \frac{\Pr[r^i = g | S = G] \Pr[S = G]}{\Pr[r^i = g]} \\ &= \frac{\theta(q + (1-q)\sigma^i)}{\theta(q + (1-q)\sigma^i) + (1-\theta)((1-q) + q\sigma^i)} \end{aligned} \tag{3}$$

Similarly, the posterior probability of G given $r^i = b$ is given by:

$$\begin{aligned} \Pr\big[S = G | r^i = b\big] &= \frac{\Pr[r^i = b | S = G] \Pr[S = G]}{\Pr[r^i = b]} \\ &= \frac{\theta(1-q)(1-\sigma^i)}{\theta(1-q)(1-\sigma^i) + (1-\theta)q(1-\sigma^i)} \\ &= \frac{\theta(1-q)}{\theta(1-q) + (1-\theta)q} \end{aligned} \tag{4}$$

After calculating the posteriors, the citizens calculate the minimum expected return that justifies investment. Recall that the cost of investing in the project is given by c. Let $\bar{X}_g^i$ be the minimum return that justifies investment following the report $r = g$. Further, let

$\bar{X}_b^i$ be the the minimum return which makes a citizen indifferent between investing and not investing following the report $r^i = b$. Therefore, $\bar{X}_g^i$ and $\bar{X}_b^i$ are respectively given by as follows:

$$\begin{aligned} \bar{X}_g^i =& \frac{c}{\Pr[S = G | r^i = g]} \\ =& c\left\{1 + \left(\frac{(1-\theta)}{\theta}\right) \frac{(1-q) + q\sigma^i}{(q + (1-q)\sigma^i)}\right\} \end{aligned} \tag{5}$$

and

$$\begin{aligned} \bar{X}_b^i =& \frac{c}{\Pr[S = G | r^i = b]} \\ =& c\left\{1 + \left(\frac{(1-\theta)}{\theta}\right) \frac{q}{1-q}\right\} \end{aligned} \tag{6}$$

This implies that any citizen with an expected return below $\bar{X}_g^i$ does not invest. Further, any citizen with an expected return greater than $\bar{X}_b^i$ always invests. Therefore, only citizens with an expected return between $\bar{X}_b^i$ and $\bar{X}_g^i$ want to subscribe to the news. Now let us consider subscription decisions. In the interval $(\bar{X}_g^i, \bar{X})$ citizens invest if news media reports $r^i = g$ but do not invest based on the prior. On the other hand, citizens with expected returns in the interval $(\bar{X}, \bar{X}_b^i)$ do not invest in the project if the media firm reports $r^i = b$ but always invest based on the prior. Let N_g net benefit from subscribing to the news in the interval $(\bar{X}_g^i, \bar{X})$ which is given by:

$$N_g = \Pr[S = G | r = g] \Pr[r = g](X - c) - \Pr[S = B | r = g] \Pr[r = g]c - d \tag{7}$$

The fist term on the right hand side of equation (7) captures the net benefit of investing in the project following the news. The second term captures the losses that accrue when the news report is inaccurate. The last term captures the dis-utility from following the news. A citizen will subscribe to the news provided that the net benefit from subscribing is non-negative, i.e., $N_g \geq 0$. Therefore, the minimum return that justifies subscribing to the news is given by:

$$\tilde{X}_g^i = c\left\{1 + \left(\frac{(1-\theta)}{\theta}\right)\frac{(1-q)+q\sigma^i}{q+(1-q)\sigma^i}\right\} + \frac{d}{\theta(q+(1-q)\sigma^i)} \tag{8}$$

Now let us consider the interval $(\bar{X}, \bar{X}_b^i)$. In this interval citizens do not invest in the project if the media firm reports $r^i = b$ and citizens always invest in the project based on the prior and earn a net expected return of $\theta X - c$. Hence, the net benefit from subscribing to the news in the $(\bar{X}, \bar{X}_b^i)$ is given by:

$$N_b = c\left\{1 + \left(\frac{(1-\theta)}{\theta}\right)\frac{q}{1-q}\right\} - \frac{d}{\theta(1-q)(1-\sigma^i)} - \theta X - c \geq 0 \tag{9}$$

Therefore, the minimum expected return that justifies subscription is given by:

$$\tilde{X}_b^i = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{q}{1-q}\right\} - \frac{d}{\theta(1-q)(1-\sigma^i)} \tag{10}$$

It is easy to show $\tilde{X}_b^i < 2m \;\forall\; q \in (\frac{1}{2}, q^0)$ for some $q^0 \in (\frac{1}{2}, 1)$.[2] Going forward, we will assume that q is bounded above by $q^0 = \frac{\left(\frac{\theta}{1-\theta}\right)\frac{2m-c}{c}}{1+\left(\frac{\theta}{1-\theta}\right)\frac{2m-c}{c}}$. Therefore, the number of people who subscribe to the news have expected returns between $[\tilde{X}_g^i, \tilde{X}_b^i]$. Therefore, the number of citizens who subscribe to the news, $\tilde{S}_i$ is given by:

$$\begin{aligned}
\tilde{S}_i =& \Pr\left[\tilde{X}_g^i \leq X \leq \tilde{X}_b^i\right] \\
=& \Pr\left[X \leq \tilde{X}_b^i\right] - \Pr\left[X \leq \tilde{X}_g^i\right] \\
=& \frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)\frac{(2q-1)}{(1-q)(q+(1-q)\sigma^i)} \\
&- \frac{d}{2\theta m(1-q)(1-\sigma^i)(q+(1-q)\sigma^i)} \\
=& \frac{1}{2\theta m(1-q)(q+(1-q)\sigma^i)}\left\{c(1-\theta)(2q-1) - \frac{d}{(1-\sigma^i)}\right\}
\end{aligned} \tag{11}$$

[2]For every $c > 0$, $\theta \in (0,1)$, $q \in (\frac{1}{2}, 1)$ $d \in \mathbb{R}_+ \cup \{0\}, \exists q^0 \in (\frac{1}{2}, 1)$. If $d = 0$ then $q^0 = \frac{\left(\frac{\theta}{1-\theta}\right)\frac{2m-c}{c}}{1+\left(\frac{\theta}{1-\theta}\right)\frac{2m-c}{c}}$.

We will assume that d is sufficiently small such that $\tilde{S}_i > 0$. Stated alternatively, $\tilde{S}_i > 0$ as long as $\tilde{X}_g^i < \bar{X}$. Since citizens in the interval $(\tilde{X}_g^i, \tilde{X}_b^i)$ subscribe to news, we must have $\tilde{S}_i \subset (\bar{X}_g^i, \bar{X}_b^i)$.

5 Informational Content under Freedom of Press

In this section, we analyze the optimal informational content of news under *freedom of press*. First, we establish that given the states of nature and the prior, the news outlet does not distort its signal $s = g$. However, it may choose to distort the signal $s = b$. In other words, the optimal editorial policy is given by $\Pr[r = g|s = b] = \sigma^P$, $\Pr[r = b|s = b] = (1 - \sigma^P)$, $\Pr[r = g|s = g] = 1$, $\Pr[r = b|s = g] = 0$. Further, we also show that $r(\sigma, \tilde{S}_P)$ can be represented by $b(\sigma^P)\tilde{S}_P^\alpha R_0$ where $b(\sigma^P) = \Pr\big[r^P = g|S = G\big] = \theta(q + (1 - q)\sigma^P)$ is the probability of predicting the good state accurately and $\tilde{S}_P^\alpha R_0$ captures the reward from doing so where $R_0 \in \mathbb{R}_+$ is a fixed dollar amount. We assume that the expected reward increases with the size of the audience/subscribers at a decreasing rate, i.e., $\alpha < 1$. Further, we will assume that R_0 is sufficiently large. We state this in result in Lemma 1.

Lemma 1: *The media firm benefits from reporting its signal $s = g$ truthfully but possibly distorting its signal $s = b$. Further, the reward from correctly predicting the actual state can be represented by $b(\sigma^P)\tilde{S}_P^\alpha R_0$ for a sufficiently large R_0.*

Proof: See Appendix A.

Comment: The intuition is straightforward. When the media firm chooses a bias in the direction of citizens' prior, the number of subscribers increases compared to the situation when the bias is in the opposite direction. Further, the reward from correctly predicting the actual state is higher when it reports the good signal correctly and possibly garbling the bad signal. Since profits increase with the number of subscribers, it makes sense for the media firm to always report the good signal truthfully and to possibly garble its bad signal.

We normalize the fixed cost of news production to zero for simplicity. Consequently, the media firm's objective function is given by:

$$\begin{aligned}\pi_P =&a\tilde{S}_P + \theta(q + (1-q)\sigma^P)\tilde{S}_P^{\alpha} R_0 \\ =&a\tilde{S}_P + b(\sigma^P)\tilde{S}_P^{\alpha} R_0\end{aligned} \tag{12}$$

Differentiating equation (12) with respect to σ^P yields:

$$\frac{\partial \pi_P}{\partial \sigma_P} = a\tilde{S}'_P + \theta(1-q)R_0\tilde{S}_P + \alpha\theta(q + (1-q)\sigma^P)\tilde{S}_P^{\alpha-1}\tilde{S}'_P R_0 \tag{13}$$

where

$$\begin{aligned}\tilde{S}'_P =& -\frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)\frac{(2q-1)}{(q+(1-q)\sigma^P)^2} \\ &- \frac{d((2q-1)(1-\sigma)+\sigma)}{2\theta m(1-q)(1-\sigma^i)(q+(1-q)\sigma^i)^2} \\ <&0\end{aligned} \tag{14}$$

Further, after some algebra we can show that $\dfrac{\partial^2 \pi_P}{\partial {\sigma^P}^2} < 0$ for a sufficiently large $R_0 \in \mathbb{R}_+$. Therefore, whether σ^{P*} is interior or not depends on the sign of $\left.\dfrac{\partial \pi_P}{\partial \sigma_P}\right|_{\sigma^P=0}$. We state our results in the following lemma.

Lemma 2: $\exists c_0 \in \mathbb{R}$, *such that* $\forall\ c < c_0$, $\sigma^{P*} = 0$ *and* $\forall\ c > c_0$, $\sigma^{P*} > 0$ *for a sufficiently large* $R_0 \in \mathbb{R}_+$.

Proof: See Appendix A.

Comment: Note that the profit of the media firm depends on two components: the size of its subscribers ($\tilde{S}_P$) and the expected reward from accurately reporting the good state ($b(\sigma^P)\tilde{S}_P^{\alpha}R_0$). The expected reward, in turn, depends on two factors: the number of subscribers ($\tilde{S}_P$) and the probability of correctly predicting the good state ($b(\sigma^P)$). Now suppose that the media firm is contemplating whether or not to increase bias when the bias is initially at zero. A rise in σ^P reduces the number of subscribers ($\tilde{S}_P$) which reduces the incentive to increase bias. Now consider the effect of a rise in σ^P on the reward from accurately reporting the good state. As σ^P rises, $b(\sigma^P)$ increases. However, $\tilde{S}_P^{\alpha}$ decreases. The net effect on expected reward depends on the relative magnitude of the two. When $c < c_0$, an increase in σ^P reduces the expected reward (i.e., the fall in $\tilde{S}_P^\alpha$ is larger than the rise in $b(\sigma^P)$). Hence, the marginal benefit from increasing bias declines. Consequently, the media firm chooses to exhibit zero bias. In other words, when the cost of investment is small the loss is also small, hence subscribing to the news is not that essential. Alternatively stated, the difference between the minimum return that justifies investment with and without the news is smaller when the cost of investment is smaller, hence the media firm mobilizes fewer people to subscribe and invest when c is smaller. As a consequence, the media firm does not distort its signal since the number of subscribers is smaller and since citizens are more sensitive to bias.

Now consider the situation when $c > c_o$. In this case, the entire argument is reversed, i.e., a rise in bias increases the expected reward from accurately predicting the good state (i.e., the fall in $\tilde{S}_P^\alpha$ is smaller than the rise in $b(\sigma^P)$). As a consequence, the marginal benefit from increasing bias rises. Hence, the media firm chooses a positive bias. In this case citizens have more to lose and hence are more tolerant of bias because, even with bias, the news is informative. Stated alternatively, the news media biases its report when citizens' losses from investment are higher, i.e., when citizens have greater preference for unbiased news..

Proposition 1: $\exists c_0 \in \mathbb{R}$ *such that* $\forall\ c < c_0$*, the news under freedom of press is most informative and* $\forall c > c_0$*, the news is least informative.*

Proof: See Appendix A.

Comment: Note that when $\sigma^P = 0$, then the media firm reports its signal truthfully. Hence, its report is most informative. However, when $\sigma^P > 0$, then it garbles only the bad signal with some probability making the news least informative.

6 Informational Content under Capture

In this section, we formalize the informational content of news under capture. Recall that the payoff the government is given by rewriting equation (2) as follows:

$$\begin{aligned} V_{Gov} =& \gamma \Pr\left[r^{Gov} = g\right] \Pr\left[\tilde{X}_g^{Gov} \leq X \leq \tilde{X}_b^{Gov}\right] \\ &+ \gamma \Pr\left[\tilde{X}_b^{Gov} \leq X \leq 2m\right] - T_{Gov} \end{aligned} \tag{15}$$

The first term of equation (15) represents the payoff the government receives when the subscribers, mobilized by the media firm, invest following a good report. The second term captures the payoff the government receives from citizens who always invest regardless of the news. The third term represents the bribe that the government pays out in *capture*. Let the likelihood of a good report be represented by the following: $n(\sigma^{Gov}) = \Pr\left[r^{Gov} = g\right] = \theta(q + (1-q)\sigma^{Gov}) + (1-\theta)((1-q) + q\sigma^{Gov})$. Further, note that the number of subscribers can be expressed as the following:

$$\begin{aligned} \tilde{S}_{Gov} =& \Pr\left[\tilde{X}_g^{Gov} \leq X \leq \tilde{X}_b^{Gov}\right] \\ =& \frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)\frac{(2q-1)}{(1-q)(q+(1-q)\sigma^{Gov})} \\ &- \frac{d}{2\theta m(1-q)(1-\sigma^{Gov})(q+(1-q)\sigma^{Gov})} \\ =& \frac{1}{2\theta m(1-q)}\frac{1}{(q+(1-q)\sigma^{Gov})}\left\{c(1-\theta)(2q-1) - \frac{d}{(1-\sigma^{Gov})}\right\} \\ =& \frac{1}{2\theta m(1-q)} h(\sigma^{Gov}) t(\sigma^{Gov}) \end{aligned} \tag{16}$$

where $h(\sigma^{Gov}) = (q+(1-q)\sigma^{Gov})^{-1}$ and $t(\sigma^{Gov}) = \left\{c(1-\theta)(2q-1) - \frac{d}{(1-\sigma^{Gov})}\right\}$. Further, let $\phi = \Pr\left[\tilde{X}_b^{Gov} \leq X \leq 2m\right] = \left(1 - \frac{c}{2m}\right) - \frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)\frac{q}{(1-q)} + \frac{d}{2\theta m(1-q)(1-\sigma^{Gov})}$ represent the number of citizens who always invest regardless of the news. Therefore, equation (15) can be rewritten as:

$$V_{Gov} = \frac{\gamma}{2\theta m(1-q)} n(\sigma^{Gov}) h(\sigma^{Gov}) t(\sigma^{Gov}) + \gamma\phi - T_{Gov} \tag{17}$$

Differentiating equation (17) with respect to σ^{Gov} yields:

$$\frac{\partial V_{Gov}}{\partial \sigma^{Gov}} = \frac{\gamma}{2\theta m(1-q)} \Bigg[\Big\{ n'(\sigma^{Gov})h(\sigma^{Gov}) + n(\sigma^{Gov})h'(\sigma^{Gov}) \Big\} t(\sigma^{Gov}) + n(\sigma^{Gov})h(\sigma^{Gov})t'(\sigma^{Gov}) + \frac{d}{(1-\sigma^{Gov})^2} \Bigg] \tag{18}$$

The government chooses the optimal bias by weighing the pros and cons of increasing bias. Specifically, a rise in bias affects the governments payoff in the following ways: on one hand, a rise in bias increases the likelihood of a good report (captured by $n(\sigma^{Gov})$). A greater likelihood, *ceteris paribus* increases the expected number of investors which creates an incentive for the government to increase bias. However, since the news is less informative when σ^{Gov} is higher, fewer people subscribe to the news outlet and therefore can be mobilized to invest. Consequently, it reduces the incentive for the government to increases bias. Finally, recall that a fraction of the citizenry always invests in the project which is captured by ϕ. A rise in σ^{Gov} increases the number of citizens who always invest. Therefore, this also creates an additional incentive to increase bias. Further, we find that V_{Gov} is strictly concave with respect to σ^{Gov}. We state the result in proposition 2.

Proposition 2: *Under capture, the government makes the news completely uninformative in equilibrium.*

Proof: See Appendix A.

Comment: As noted above, a rise in bias increases the likelihood of receiving a good report which increases the expected number of investors. Further, a rise in bias also increases the number of people who always invest. These two forces create an incentive for the government to increase bias.[3] However, a rise in bias also reduces the number of citizens who can be mobilized to invest by the news because the news is now less informative. This creates a disincentive for the government to increase bias. However, we find that when signal quality is below a certain threshold, the loss in the number of investors due to a rise in bias is smaller because the news mobilizes fewer citizens in the first place.

[3] When $q \geq q^0$, the number of people who always invest goes to zero. Consequently, the government's incentive to increase bias is diminished in that case.

Hence, the first two forces dominate the last. Hence, the government keeps increasing bias until the news becomes completely uninformative. Stated alternatively, recall that without the news, every citizen with an expected return above $\bar{X}$ always invests in the project based on their prior (i.e., in the absence of news). We find that when signal quality is below q^0, the expected number of citizens who invest when the news is available is smaller than the expected number of citizens who invest based on the prior (i.e., in the absence of news) as long as $\sigma^{Gov} < 1$ (they are equal when $\sigma^{Gov} = 1$). Hence, the government makes the news completely uninformative in order to maximize the expected number of investors. Consequently, citizens stop following the news and make investment choices based on the prior.

Proposition 3: *Capture becomes more attractive to the government as the cost of investment or mobilization decreases or the dis-utility from news increases.*

Proof: See Appendix A.

Comment: Note that under capture the news becomes completely uninformative. Since no one subscribes to the news in that case, the media firm does not earn any profit. Hence, the government must compensate the media firm for the net loss of profit which is $\pi|_{\sigma^{P*}}$, therefore, $T_{Gov} = \pi|_{\sigma^{P*}}$. Now consider what happens to the firm's profit as the cost of investment falls. Recall that citizens have less to lose when investments are less costly. Hence, the number of subscribers decreases following a fall in c, holding everything else constant. Since the profit of the firm increases monotonically with the number of subscribers, the firm's profit decreases. Therefore, *capture* becomes more attractive as the cost of investment decreases. The intuition for an increase in the dis-utility from following the news is similar.

7 Conclusion

In this paper, we show that when the quality of the signal is below a certain threshold, then the media firm's report (under *freedom of press*) is most informative when the cost of investment is below a certain threshold and least informative when the cost of investment is above that threshold. In contrast, the media firm's report (under *capture*) is completely uninformative to the point that citizens no longer subscribe to or follow the news. The government does this to mobilize the maximum number of citizens to invest in its desired project. We further show that capture becomes more attractive for the government, *ceteris paribus*, when the dis-utility from following the news is higher. On the other hand, capture becomes less attractive from the point of view of the government as the cost of investment increases.

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A Appendix A: Proofs of lemmas and propositions

Lemma 1: *The media firm benefits from reporting its signal $s = g$ truthfully but possibly distorting its signal $s = b$. Further, the reward from correctly predicting the actual state can be represented by $b(\sigma^P)\tilde{S}_P^{\alpha} R_0$ for a sufficiently large R_0.*

Proof: We prove lemma 1 by proving three claims. Consider the following editorial policies:

Editorial policy A:

$\Pr[r = g|\ s = b] = \sigma^P$, $\Pr[r^P = b|\ s = b] = (1 - \sigma^P)$, $\Pr[r^P = g|\ s = g] = 1$, $\Pr[r^P = b|\ s = g] = 0$ and

Editorial Policy B:

$\Pr[r^P = g|\ s = b] = 0$, $\Pr[r^P = b|\ s = b] = 1$, $\Pr[r^P = g|\ s = g] = 1 - \hat{\sigma}^P$, $\Pr[r^P = b|\ s = g] = \hat{\sigma}^P$.

Claim 1: *The expected number of subscribers is higher under editorial policy A.*

Proof: Note that the good signal is reported truthfully under editorial policy A whereas the bad signal is reported truthfully under editorial policy B. Since citizens only gain from investing when the actual state is good, they calculate the posteriors using Bayes' theorem. Therefore, the posterior probability of the good state given report $r = b$ is given by:

$$\begin{aligned}\Pr\big[S = G|r^P = b\big] =& \frac{\Pr\big[r^P = b|S = G\big]\Pr[S = G]}{\Pr[r^P = b]},\\ =& \frac{\theta\{(1-q) + q\hat{\sigma}^P\}}{\theta\{(1-q) + q\hat{\sigma}^P\} + (1-\theta)\{q + (1-q)\hat{\sigma}^P\}}.\end{aligned} \tag{A.1}$$

Let $\hat{X}_b^P$ denote the minimum return required to justify investment following $r^P = b$ under editorial policy B. Therefore,

$$\hat{X}_b^P = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{q + (1-q)\hat{\sigma}^P}{(1-q) + q\hat{\sigma}^P}\right\}. \tag{A.2}$$

Note that $\hat{X}_b^P < \bar{X}_b = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{q}{1-q}\right\}$. Similarly, the posterior probability of the good state given a report $r = g$ under editorial policy B is given by the following:

$$\begin{aligned}\Pr\big[S = G|r^P = g\big] =& \frac{\Pr\big[r^P = g|S = G\big]\Pr[S = G]}{\Pr[r^P = g]},\\ =& \frac{\theta q}{\theta q + (1-\theta)(1-q)}.\end{aligned} \tag{A.3}$$

As before, let $\hat{X}_g^P$ denote the minimum return to justify investment under editorial policy B and is given by:

$$\hat{X}_g^P = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{1-q}{q}\right\}. \tag{A.4}$$

Note that $\hat{X}_g^P < \bar{X}_g = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{(1-q)+q\sigma^P}{q+(1-q)\sigma^P}\right\}$ for $\forall\sigma^P$. Now, consider the intervals $(\hat{X}_g^P, \bar{X})$ and $(\bar{X}, \hat{X}_b^P)$ under editorial policy B. In the interval $(\hat{X}_g^P, \bar{X})$ citizens do not invest based on the prior but invest in the project following $r^P = g$. In the interval $(\bar{X}, \hat{X}_b^P)$, citizens always invest based on the prior but do not invest following $r^P = b$. Therefore, the net benefit, N_g^P, of subscribing to the media outlet in the interval $(\hat{X}_g^P, \bar{X})$ is given by the following:

$$\begin{aligned}N_g^P =& \Pr\big[r^P = g\big]\Pr[S = G|r = g](X - c) - \Pr\big[r^P = g\big]\Pr\big[S = B|r^P = g\big]c\\ =& \theta q(1-\hat{\sigma}^P)(X - c) - (1-\theta)c(1-q)(1-\hat{\sigma}^P)\end{aligned} \tag{A.5}$$

Recall that the cost of following news is captures by d. As a result, the minimum return required from the investment to justify subscribing to the news is given by

$$\tilde{\hat{X}}_g^P = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{1-q}{q}\right\} + \frac{d}{\theta q(1-\hat{\sigma}^P)}. \tag{A.6}$$

Similarly, the minimum return to justify subscribing to news in the interval $(\bar{X}, \hat{X}_b^P)$ is given by:

$$\tilde{\hat{X}}_b^P = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{q+(1-q)\hat{\sigma}^P}{(1-q)+q\hat{\sigma}^P}\right\} - \frac{d}{\theta((1-q)+q\hat{\sigma}^P)}. \tag{A.7}$$

Since anyone with an expected return less than $\hat{X}_g^P$ does not invest in the project, they do not have any need to subscribe to the media outlet. Similarly, any citizen with a return above $\hat{X}_b^P$ always invests in the project, they also do not subscribe to the news. Therefore, the expected number of subscribers under editorial policy B are citizens whose expected return lies in the interval $\tilde{\hat{X}}_b^P - \tilde{\hat{X}}_g^P$ which after a little algebra yields:

$$\tilde{\hat{S}}_P = \left\{\frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)(2q-1) - \frac{d}{2m\theta(1-\hat{\sigma}^P)}\right\}\frac{1}{q((1-q)+q\hat{\sigma}^P)} \tag{A.8}$$

Now consider the expected number of subscribers under editorial policy A given that the magnitude of the bias is the same under both editorial policies given by the following:

$$\hat{S}_P = \left\{\frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)(2q-1) - \frac{d}{2m\theta(1-\hat{\sigma}^P)}\right\}\frac{1}{(1-q)(q+(1-q)\hat{\sigma}^P)} \tag{A.9}$$

Since $\dfrac{1}{q((1-q)+q\hat{\sigma}^P)} < \dfrac{1}{(1-q)(q+(1-q)\hat{\sigma}^P)}$ for all $q >\in (\frac{1}{2}, 1)$, it must be that $\hat{S}^P > \tilde{\hat{S}}^P$. Hence proved ■

Claim 2: *The return from correctly predicting the actual state is higher under editorial policy A provided that the ratio of return from predicting the good state and the bad state correctly is at least as high as* $\dfrac{1-\theta}{\theta}$.

Proof: Let R_G and R_B represent the return from accurately predicting the good and the bad state respectively. We will assume that R_G is sufficiently large compared to R_B. This may be reflect the fact that citizens stand to gain more on average $(m > c)$ when the good state is predicted accurately relative to the bad state (recall that if the bad state is predicted accurately citizens, who avoid investing, only save c). Therefore, the reward from correctly predicting the actual state, under editorial policy A, is given by:

$$\begin{aligned} r(\sigma^P) =& \Bigg\{ \Pr[r = g|S = G] \Pr[S = G] R_G + \Pr[r = b|S = B] \Pr[S = B] R_B \Bigg\} \tilde{S}_P^\alpha \\ =& \Bigg\{ \theta(q + (1-q)\sigma^P) R_G + (1-\theta)(1-\sigma^P) q R_B \Bigg\} \tilde{S}_P^\alpha \end{aligned} \tag{A.10}$$

Similarly, the reward from accurately predicting the actual state under editorial policy B is given by:

$$\begin{aligned} \hat{r}(\hat{\sigma}^P) =& \Bigg\{ \Pr[r = b|S = B] \Pr[S = B] R_B + \Pr[r = g|S = G] \Pr[S = G] R_G \Bigg\} \tilde{S}_P^\alpha \\ =& \Bigg\{ (1-\theta)(q + (1-q)\hat{\sigma}^P) R_B + \theta(1-\hat{\sigma}^P) q R_G \Bigg\} \tilde{S}_P^\alpha \end{aligned} \tag{A.11}$$

Suppose that $\sigma^P = \hat{\sigma}^P$. Clearly, $r(\sigma^P) > \hat{r}(\hat{\sigma}^P)$ $\forall R_G \in \mathbb{R}_+$ such that $\dfrac{R_G}{R_B} \geq \dfrac{1-\theta}{\theta}$ for every $R_B \in \mathbb{R}_+$ and for any $\theta \in (0,1)$, since

$$r(\hat{\sigma}^P) - \hat{r}(\hat{\sigma}^P) = \hat{\sigma}^P \Bigg\{ \theta R_G - (1-\theta) R_B \Bigg\} \tag{A.12}$$

Given the profit function is monotonic in the number of subscribers (equation (1) in the main text), the profit under editorial policy A is higher than than that under B. Further, $\hat{\sigma}^P$ is not the equilibrium bias under editorial policy A, therefore, the profit under editorial policy A must be higher in equilibrium. Hence proved ■

Claim 3: For every $R_B \in \mathbb{R}_+$ we can find a sufficiently large $R_G = R_0 \in \mathbb{R}_+$ such that $r(\sigma^P) = R_0 b(\sigma^P) \tilde{S}_P^\alpha$.

Proof: For a sufficiently large $R_G = R_0$ compared to R_B, we must have the following:

$$\begin{aligned} r(\sigma^P) =& R_G \Bigg\{ \theta(q + (1-q)\sigma^P) + (1-\theta)(1-\sigma^P) q \frac{R_B}{R_G} \Bigg\} \tilde{S}_P^\alpha \\ \approx& R_G \theta(q + (1-q)\sigma^P) \tilde{S}_P^\alpha \\ =& b(\sigma^P) \tilde{S}_P^\alpha R_0 ■ \end{aligned} \tag{A.13}$$

Lemma 2: *$\exists c_0 \in \mathbb{R}$, such that $\forall\ c < c_0$, $\sigma^{P*} = 0$ and $\forall\ c > c_0$, $\sigma^{P*} > 0$ for a sufficiently large $R_0 \in \mathbb{R}_+$.*

Proof: Note that whether or not we have an interior optimum depends on the sign of $\left.\frac{\partial \pi_P}{\partial \sigma^P}\right|_{\sigma^P=0}$. After a little algebra we can show that:

$$\left.\frac{\partial \pi_P}{\partial \sigma^P}\right|_{\sigma^P=0} = aS'(0) + b'(0)S^{\alpha}(0)R_0 + b(0)\alpha S(0)^{\alpha-1}S'(0)R_0 \tag{A.14}$$

It is easy to show that $aS'(0) < 0$. Now consider the term F, given by:

$$\begin{aligned} F =& S(0)^{\alpha-1}\Big[b'(0)S(0) + \alpha b(0)S'(0)\Big]R_0 \\ =& S(0)^{\alpha-1}R_0\varphi \end{aligned} \tag{A.15}$$

where the bracketed expression, φ, can be shown to be equal to the following:

$$\varphi = \frac{c}{2m}\frac{(1-\theta)(2q-1)(1-\alpha)}{q} - \frac{d\left(1 + \frac{(2q-1)\alpha}{(1-q)}\right)}{2mq} \tag{A.16}$$

It is to easy that φ monotonically increasing in c. Therefore, $\exists c_0 \in \mathbb{R}$ such that $\forall\ c < c_0$, $\varphi < 0$ and $\forall\ c > c_0$, $\varphi > 0$. Therefore, $\forall\ c < c_0$, $\sigma^{P*} = 0$. However, for a sufficiently large $R_0 \in \mathbb{R}_+$, $\sigma^{P*} > 0$ since $\varphi > 0\ \forall\ c > c_0$. Finally, note that $\tilde{S}_P(1) = 0$. Since the profit function of the media firm under *freedom of press* is strictly concave with respect to σ^P, we must have the desired result ■

Proposition 1: *$\exists c_0 \in \mathbb{R}$ such that $\forall\ c < c_0$, the news under freedom of press is most informative and $\forall c > c_0$, the news is least informative.*

Proof: Combining Lemma 1 and Lemma 2 completes the proof ■

Proposition 2: *Under capture, the government makes the news completely uninformative in equilibrium.*

Proof: Note that from equation (18), we only need to show that $t'(\sigma^{Gov}) > 0$ and $n'(\sigma^{Gov})h(\sigma^{Gov}) + n(\sigma^{Gov})h'(\sigma^{Gov}) > 0$. It is easy to check that $t'(\sigma^{Gov}) = \frac{d}{(1-\sigma^{Gov})^2} > 0$. Further, with a little bit of algebra, we can show that $n'(\sigma^{Gov})h(\sigma^{Gov}) + n(\sigma^{Gov})h'(\sigma^{Gov}) =$

$\frac{(1-\theta)(2q-1)}{(q+(1-q)\sigma^{Gov})^2} > 0$. As a result, we must have $\frac{\partial V_{Gov}}{\partial \sigma^{Gov}} > 0$. Further, we can show that $\frac{\partial^2 V_{Gov}}{\partial {\sigma^{Gov}}^2} = \frac{-\gamma(1-\theta)^2(2q-1)^2 c}{2\theta m(q+(1-q)\sigma^{Gov})^3} < 0$, i.e., V_{Gov} is strictly concave with respect to σ^{Gov}. Hence, we must have $\sigma^{Gov*} = 1$. However, when $q \geq q^0$, $\tilde{X}_b \geq 2m$. Hence, the number of people who always invest is zero. In that case, $\tilde{S}_{Gov} = 0$ at $\sigma^{Gov} = 1$ which implies that $V_{Gov}(1) = 0$, therefore, $\sigma^{Gov*} < 1$ in that case. Consequently, $V_{Gov}|_{\sigma^{Gov*}=1} > 0$ only when $q < q^0$. Hence proved ■

Proposition 3: *Capture becomes more attractive to the government as the cost of investment or mobilization decreases or the dis-utility from news increases.*

Proof: Recall that under capture, $\sigma^{Gov*} = 1$. Hence, $\tilde{S}_P(1) = 0$. Therefore, the profit of the firm from aligning with the government is 0. As a consequence, the government must compensate for the loss of profit. This implies that:

$$T_{Gov} = \pi_P \Big|_{\sigma^{P*}} \tag{A.17}$$

Using the Envelope Theorem, we have $\frac{\partial T_{Gov}}{\partial c} = \frac{\partial \pi_P|_{\sigma^{P*}}}{\partial c}\cdot$

Recall that $\tilde{S}_P = \frac{1}{2\theta m(1-q)(q+(1-q)\sigma^P)}\left\{c(1-\theta)(2q-1) - \frac{d}{(1-\sigma^P)}\right\}$. It is easy to show that $\frac{\partial \tilde{S}_P}{\partial c} > 0$. Since the profit function is monotonic in $\tilde{S}_P$,we must have $\frac{\partial T_{Gov}}{\partial c} > 0\cdot$

The proof for $\frac{\partial T_{Gov}}{\partial d}$ is similar. Since the government's net pay-off depends on the magnitude of the bribe, the result follows ■

B Appendix B: Endogenous Subscription Price

Let us now consider the scenario where citizens pay a price to subscribe to the media outlet. Let the price of subscription be denoted by p^i for $i \in \{P, Gov\}$. The minimum return required to subscribe to the news in the interval $(\bar{X}_g, \bar{X})$ is given by:

$$\tilde{X}_g^i = c\left\{1 + \left(\frac{(1-\theta)}{\theta}\right)\frac{q}{1-q}\right\} + \frac{p^i + d}{\theta(q + (1-q)\sigma^i)} \tag{B.1}$$

Similarly, the minimum return required to justify subscribing to the news in the interval $(\bar{X}, \bar{X}_b)$ is given by:

$$\tilde{X}_b = c\left\{1 + \left(\frac{1-\theta}{\theta}\right)\frac{q}{1-q}\right\} - \frac{p^i + d}{\theta(1-q)(1-\sigma^i)} \tag{B.2}$$

Therefore, the number of citizens who are expected to subscribe to the news is given by:

$$\begin{aligned}
\tilde{S}_i =& \Pr\left[\tilde{X}_g^i \le X \le \tilde{X}_b^i\right] \\
=& \Pr\left[X \le \tilde{X}_b^i\right] - \Pr\left[X \le \tilde{X}_g^i\right] \\
=& \frac{c}{2m}\left(\frac{1-\theta}{\theta}\right)\frac{(2q-1)}{(1-q)(q+(1-q)\sigma^i)} \\
&- \frac{p^i + d}{2\theta m(1-q)(1-\sigma^i)(q+(1-q)\sigma^i)} \\
=& \frac{1}{2\theta m(1-q)(q+(1-q)\sigma^i)}\left\{c(1-\theta)(2q-1) - \frac{p^i + d}{(1-\sigma^i)}\right\}
\end{aligned} \tag{B.3}$$

From here, by repeating the steps of lemma 1 we can show that the profit function of the firm can now be written as:

$$\pi_i = (p^i + a)\tilde{S}_i + b(.)\tilde{S}_i^{\alpha} R_0 + T_i \tag{B.4}$$

The firm first determines the optimal price to charge in equilibrium which is obtained by differentiating equation (B.2) with respect to p^i which yields:

$$p^{i*} = \frac{c}{2}(1-\theta)(2q-1)(1-\sigma^i) - \frac{a+d}{2} \tag{B.5}$$

It is important to note that p^{i*} is bounded below by 0. Further, plugging the expression for p^{i*} in equation (B.3) we get:

$$\begin{aligned}\tilde{S}_i =& \Pr\left[\tilde{X}_g^i \leq X \leq \tilde{X}_b^i\right] \\ =& \Pr\left[X \leq \tilde{X}_b^i\right] - \Pr\left[X \leq \tilde{X}_g^i\right] \\ =& \frac{c}{4m}\left(\frac{1-\theta}{\theta}\right)\frac{(2q-1)}{(1-q)(q+(1-q)\sigma^i)} \\ &+ \frac{(a-d)}{4\theta m(1-q)(1-\sigma^i)(q+(1-q)\sigma^i)}\end{aligned} \tag{B.6}$$

Once we obtain the expression for $\tilde{S}_P$ and $\tilde{S}_{Gov}$ by endogenizing subscription prices, the steps in the proofs for lemma 2 and proposition 1, 2 and 3 (in appendix A) can be repeated exactly along the same lines to yield the required results as laid out in lemmas 2 and propositions 1, 2 and 3 (detailed proofs can be made available on request).