The refugee game: Blame thy neighbor for terrorism

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Abstract: This paper studies a three player hierarchical differential game (with a Large country, a Small country, and terrorist organization), to analyze the actual European refugee situation. Terrorists may enter Europe as refugees, taking advantage of the Open Doors Policy, to attack both countries. There are two scenarios: myopia and full awareness. Countries are myopic when they ignore each other's security efforts, and the terrorist group only considers the weakest link's security efforts. A comparison between the scenarios shows that for an extremely impatient Large country, full awareness yields a greater level of security effort for the Large country, a greater level of security effort for the Small country, and, because of backlash, more terrorist attacks. One predicable result is that both countries will blame each other for their failures in curbing terrorist activities. Stability analysis, however, shows that this is an unstable equilibrium. Continental safety is higher in the myopic model than in the full awareness model.

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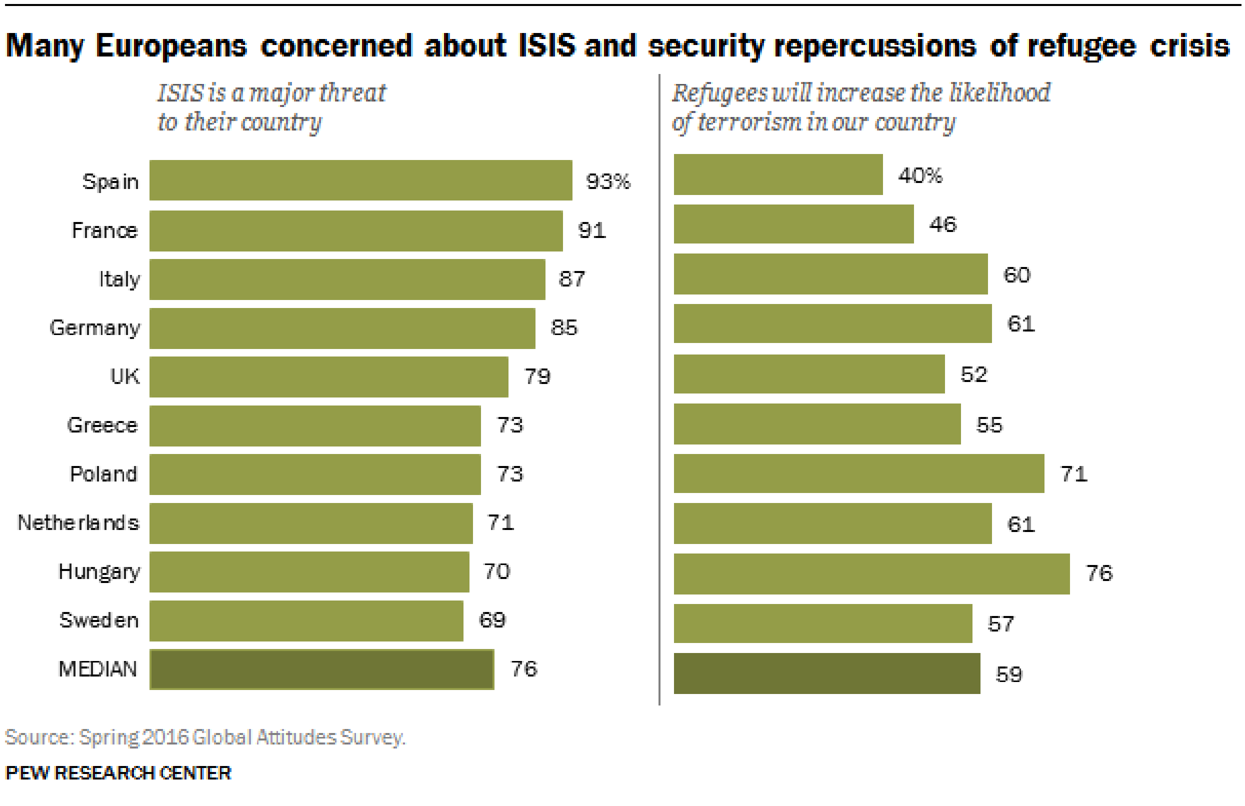
1. Introduction

In August 2015, German Chancellor Angela Merkel announced her new Open Doors Policy [ODP] "to provide refuge to anyone coming from Syria in addition to others seeking protection from violence and warfare" [Der Spiegel, 21 Sept., 2015, http://www.spiegel.de/international/germany/refugee-policy-of-chancellor-merkel-divides-europe-a-1053603.html]. A few months later, on November 13, 2015, Paris suffered terror attacks that killed at least 130 people and wounded hundreds [CNN, 30 Nov., 2015, http://www.cnn.com/2015/12/08/europe/2015-paris-terror-attacks-fast-facts/]. There is evidence that some of the Paris attackers traveled to Syria [BBC, 27 April, 2016, http://www.bbc.com/news/world-europe-34832512], raising concerns that ISIS was taking advantage of the ODP to infiltrate its operatives to organize, lead, and carry on terrorist attacks in Europe. In 2016, officials at Frontex, EU's border and coast guard agency, admitted that ISIS terrorists are taking advantage of shambolic border checks by posing as migrants to infiltrate Europe [Sunday Express, April 6, 2016, http://www.express.co.uk/news/world/658508/EU-migrant-crisis-Islamic-State-ISIS-refugees-Syria-Greece-Italy-terror-Paris-attacks].

Not surprisingly, most Europeans fear ISIS terrorist attacks and think that refugees will increase the likelihood of terrorist attacks in their countries, as showed on December

6, 2016 by the PEW Research Center graph [http://www.pewresearch.org/fact-tank/2016/12/06/4-factors-driving-anti-establishment-sentiment-in-europe/ft_16-12-06_europeconcerns_security/] below:



This paper puts forward a hierarchical differential game as a representation of the actual European refugee situation. In the model, there are three agents, two countries – one Large, which is the leader, and one Small – and one terrorist organization. Terrorists may enter Europe as refugees, taking advantage of the ODP, to attack both countries. We analyze two possible scenarios. In the first scenario, both countries are myopic and ignore each other's security efforts, and the terrorist group only considers the weakest link's security effort, which is the Small country. In the second scenario, there is full awareness by all agents. Both countries take each other's security efforts into consideration, and the terrorist group also pays attention to the Large country's security efforts. A comparison between the scenarios yields the result that for an extremely impatient Large country, full awareness yields a greater level of security effort for the Large country, a greater level of security effort for the Small country, and, because of backlash [e.g., Faria and Arce, 2005, 2012a; Siqueira and Sandler, 2007], more terrorist attacks. Despite greater security efforts, terrorist attacks escalate. One predicable result is that both countries will blame each other for their failures in curbing terrorist activities. We show, however, that this equilibrium is unstable. The best scenario for the continental safety as a whole is the myopic scenario.

The paper is structured as follows. The next section presents a literature review. The differential game is specified in Section three. Section four presents the stability analysis, and the comparison between the scenarios is presented in Section five. Concluding remarks appear in Section six.

2. Literature Review

Studies on the link between refugees and political violence are few and recent. It has been recognized that refugees are victims of violence, but they can also spread conflict and instability to their host countries. Lischer (2005), Salehyan and Gleditsch (2006) and Buhaug and Gleditsch (2008) study the link between refugees and the spread of civil war.

Salehyan (2008) examines the negative externalities for receiving states associated with refugee flows, and shows how refugee flows between states significantly increase the likelihood of militarized interstate disputes. When rebel organizations find sanctuaries in neighboring countries, insurgencies give rise to disputes between states. Salehyan (2009) studies the cases of the Nicaraguan and Rwandan civil wars, as well as Kurdish PKK and Taliban fighters in Pakistan.

The issue of terrorist organizations finding sanctuary in other countries was analyzed by Piazza (2008) and Barros et al. (2007, 2008). Piazza (2008) finds that failed states are statistically more likely to hold terrorist groups that commit transnational attacks. The articles by Barros et al. analyze terrorist attacks targeting US citizens in Europe and Africa. Their main finding is that terrorist attacks are persistent. Barros et al. (2008) presents a model that bridges the civil war literature [e.g., Murdoch and Sandler, 2004; Collier and Hoeffler, 2004,] with terrorist activities, where conflict-ridden countries become safe havens for terrorists. Related, but in some respects in the opposite direction, Lowenberg and Mathews (2008) argue that a target country like the US may find it optimal to station forces near a terrorist's base of operations as an "easy target" in order to divert attacks away from domestic targets within their homeland.

Only recently has the link between refugees and terrorism received proper scientific attention. Choi and Salehyan (2013) examine a database of 154 countries from 1970 through 2007. They show that countries with high numbers of refugees are more likely to experience both domestic and international terrorism. A natural policy recommendation is that humanitarian efforts must be coupled with robust security provisions.

Analytically, terrorism has been modeled in dynamic settings by several authors using a plethora of frameworks[1] such as systems of differential and difference equations, optimal control, dynamic programming, and differential-difference equations, [e.g., Feichtinger et al. (2001); Udwadia et al. (2006); Caulkins et al. (2008); Jensen (2011); Faria and Arce (2012b)]. [2]

The use of differential games as a tool to analyze terrorism is still in its infancy. Most of the models analyze a differential game between a government fighting a terrorist organization. Faria (2003) generates terror cycles with the government as Stackelberg leader. In Feichtinger and Novak (2008) the differential game is simultaneous and both agents are subjected to the dynamics of terrorist resources, characterized by a non-linear growth function. In a related framework, Novak et al. (2010) consider a state-separable differential game and determine Nash and Stackelberg solutions in analytic form. They show that, for specific parameter values, the government acts more aggressively and terrorists more cautiously in the Stackelberg equilibrium compared to the Nash case.

[1] For surveys see, e.g., Bueno de Mesquita (2008) and Intriligator (2010).

[2] New approaches include integral equations [e.g., Faria (2011)] and stochastic models such as queue theory and optimal stopping approaches [see Kaplan (2010) and Jensen (2016)].

The contribution of the present paper is to provide a differential game model to study the link between refugees and terrorism considering externalities between countries with respect to provision of security.

3. Differential Game

Consider a game with three players: a terrorist group plus two European countries, Small and Large. The terrorist group may use operatives under the pretense of refugees to more easily penetrate Europe, taking advantage of the Open Doors Policy [ODP]. They aim at attacking both countries by exerting effort A. The Large and the Small countries act to guarantee their own security, with L and G respectively denoting chosen levels of security efforts. The Large country is the Stackelberg leader in the game. The public safety, denoted S, of the continent depends positively on G and L and negatively on A.

We present and analyze two scenarios. In the first scenario all agents are myopic, both countries ignore each other's security measures, and the terrorist group attacks the weakest link, so it targets the Small country security. In the second scenario all agents are aware of each other, both countries pay attention to each other's security measures, and the terrorist group pays attention equally to both countries.

3.1 First Scenario: Myopia, Countries Ignore Each Other

As followers in the differential game, the Small country plays against the terrorist group. Small country welfare is positively affected by its own security efforts, *G*, and is negatively affected by terrorist's activities, *A*. Terrorist's benefits increase in *A* but decrease in *G* and *S*. Terrorists are only paying attention to the security efforts of the Small country because they see it as the weakest link [see Enders and Sandler, 2012, pp. 126-129].

Each player maximizes the present value of its benefits net of costs, given by the following objective functions. The small country's objective function is

$$\underset{G}{Max}\int_0^\infty [a_1(G-0.5G^2)-a_2A]e^{-\delta t}dt, \quad (1)$$

and the terrorist group's objective function is

$$\underset{A}{Max}\int_0^\infty [b_1(A-0.5A^2)-b_2SG]e^{-rt}dt. \quad (2)$$

In the objective functions benefits are linear and costs are convex, consistent with constant marginal benefits and increasing marginal costs, which are sufficient conditions to guarantee optimality of control choices. Note that δ and *r* are respectively the rate of time preference of Small country and terrorist group. These two decision makers face convex adjustment costs associated with their own activity. Further, the terrorist group is negatively impacted by the combination of Small country security efforts and the level of public safety in Europe, while Small country is negatively impacted by terrorist group activities.

The state equation that describes the evolution over time of public safety in Europe, S, is given by,

$$\dot{S} = \gamma L + \alpha G - \beta A - \sigma S - \Psi\,, \tag{3}$$

where the initial value of $S(0)$ is given, and L is Large country's security effort. Public safety grows over time with Large and Small countries' security efforts, L and G, and is reduced by terrorist activity A. All parameters of the model are positive.

The Small country Hamiltonian is,

$$H_G = [a_1(G - 0.5G^2) - a_2 A] + \lambda_1[\gamma L + \alpha G - \beta A - \sigma S - \Psi], \tag{4}$$

And the terrorists' Hamiltonian is,

$$H_A = [b_1(A - 0.5A^2) - b_2 SG] + \lambda_2[\gamma L + \alpha G - \beta A - \sigma S - \Psi]. \tag{5}$$

Maximizing the Hamiltonians with respect to the control variables yields,

$$\frac{\partial H_G}{\partial G} = 0 \Rightarrow a_1(1 - G) + \lambda_1 \alpha = 0 \Rightarrow G = 1 + \frac{\alpha \lambda_1}{a_1}, \tag{6}$$

and

$$\frac{\partial H_A}{\partial A} = 0 \Rightarrow A = 1 - \frac{\beta \lambda_2}{b_1}. \tag{7}$$

The adjoint equations are,

$$\dot{\lambda}_1 - \delta \lambda_1 = -\frac{\partial H_G}{\partial S} \Rightarrow \dot{\lambda}_1 = (\delta + \sigma)\lambda_1, \tag{8}$$

and

$$\dot{\lambda}_2 - r\lambda_2 = -\frac{\partial H_A}{\partial S} \Rightarrow \dot{\lambda}_2 = (r + \sigma)\lambda_2 + b_2 G, \tag{9}$$

wherein the usual transversality conditions apply. Differentiating (6) and (7) with respect to time and substituting into (8) and (9) yields the dynamic reaction functions, the best dynamic reply functions, of Small country and the terrorist organization,

$$\dot{G} = (\delta + \sigma)(G - 1), \tag{10}$$

and

$$\dot{A} = (r + \sigma)(A - 1) - \frac{\beta b_2}{b_1} G. \tag{11}$$

As expected, (10) and (11) reflect how the Small country and the terrorist organization affect each other. It is important to stress that these best response functions do not take into explicit account the role of the Large country security effort, *L*.

The Large country as the leader considers the dynamic reaction function of the Small country and terrorists, i.e., equations (10) and (11), as well as the state equation (3), in maximizing the objective function,

$$\underset{L}{Max} \int_0^\infty [\xi(L - 0.5L^2) - \phi A] e^{-Rt} dt, \text{ subject to (3), (10) and (11),} \tag{12}$$

where *R* is the Large country rate of time preference. In the Large country objective function (12), there are convex (quadratic) adjustment costs associated with the level of its own security effort and a negative effect of terrorists' actions.

The current-value Hamiltonian of the Large country problem is

$$H_L = [\xi(L - 0.5L^2) - \varphi A] + \lambda_L[\gamma L + \alpha G - \beta A - \sigma S - \Psi] + \mu_1\left[(\delta + \sigma)(G - 1)\right] + \mu_2\left[(r + \sigma)(A - 1) - \frac{\beta b_2 G}{b_1}\right] \quad , \quad (13)$$

where λ , μ_1, and μ_2 are respectively the Large country shadow prices of public safety, Small country's action, and terrorist's action. The first-order condition and adjoint equations are:

$$\frac{\partial H_L}{\partial L} = 0 \Rightarrow \xi(1 - L) + \lambda_L \gamma = 0 \Rightarrow L = 1 + \frac{\lambda_L \gamma}{\xi}, \quad (14)$$

$$\dot{\lambda}_L - R\lambda_L = -\frac{\partial H_L}{\partial S} \Rightarrow \dot{\lambda}_L = (R + \sigma)\lambda_L, \quad (15)$$

$$\dot{\mu}_1 - R\mu_1 = -\frac{\partial H_L}{\partial G} \Rightarrow \dot{\mu}_1 = R\mu_1 - \left\{\alpha\lambda_L + \mu_1(\delta + \sigma) - \mu_2 \frac{\beta b_2}{b_1}\right\}, \quad (16)$$

and

$$\dot{\mu}_2 - R\mu_2 = -\frac{\partial H_L}{\partial A} \Rightarrow \dot{\mu}_2 = R\mu_2 - \left\{-\phi - \beta\lambda_L + \mu_2(r + \sigma)\right\}. \quad (17)$$

From Eq. (15) the steady state equilibrium value of the Large country shadow price of public safety, λ_L, is,

$$\dot{\lambda}_L = 0 \Rightarrow \lambda_L = 0. \quad (15')$$

Note that, as a consequence of (15′), we have from (14) that the steady state equilibrium value of the Large country security, $\overline{L}$, is constant and unitary:

$$\overline{L} = 1. \quad (14')$$

It then follows from equation (10) that the steady state value of Small country security $\overline{G}$ is also constant and unitary:

$$\dot{G} = 0 \Rightarrow \overline{G} = 1, \tag{10'}$$

Substituting the steady state value of $\overline{G}$ into (11) yields the steady state value of terrorist organization's actions:

$$\dot{A} = 0 \Rightarrow \overline{A} = 1 + \frac{\beta b_2}{b_1(r+\sigma)}. \tag{18}$$

Substituting (10′), (14′), and (18) into (3) yields the steady state value of public safety in Europe:

$$\dot{S} = 0 \Rightarrow \overline{S} = \sigma^{-1}\left[\gamma + \alpha - \beta\left(1 + \frac{\beta b_2}{b_1(r+\sigma)}\right) - \Psi\right], \tag{19}$$

3.2 Second Scenario: Countries Consider Each Other

Now we consider a full awareness model. Countries know that their expenditure on safety have a positive externality on the safety of their neighbors and on the whole continent.

The Small country problem is:

$$\underset{G}{Max}\int_0^{\infty}[a_1(G-0.5G^2) - a_2A + a_3L + a_4S]e^{-\delta t}dt, \tag{20}$$

Note that, in contrast to equation (1), the objective function captures the positive impact of Large country and continental safety measures.

The Terrorist group problem is:

$$\underset{A}{Max}\int_0^{\infty}[b_1(A-0.5A^2)-b_2S(G+L)]e^{-rt}dt\,, \tag{21}$$

In contrast with equation (2), the terrorist group is now aware of the security actions of both countries.

The state equation that describes the evolution over time of public safety in Europe, *S*, is the same as in equation (3),

$$\dot{S}=\gamma L+\alpha G-\beta A-\sigma S-\Psi\,. \tag{22}$$

The Small country Hamiltonian is,

$$H_G=[a_1(G-0.5G^2)-a_2A+a_3L+a_4S]+\lambda_1[\gamma L+\alpha G-\beta A-\sigma S-\Psi]\,, \tag{23}$$

and the terrorists' Hamiltonian is,

$$H_A=[b_1(A-0.5A^2)-b_2S(G+L)]+\lambda_2[\gamma L+\alpha G-\beta A-\sigma S-\Psi]\,. \tag{24}$$

Maximizing the Hamiltonians with respect to the control variables yields,

$$\frac{\partial H_G}{\partial G}=0\Rightarrow a_1(1-G)+\lambda_1\alpha=0\Rightarrow G=1+\frac{\alpha\lambda_1}{a_1} \tag{25}$$

and

$$\frac{\partial H_A}{\partial A}=0\Rightarrow A=1-\frac{\beta\lambda_2}{b_1}\,. \tag{26}$$

The adjoint equations are

$$\dot{\lambda}_1-\delta\lambda_1=-\frac{\partial H_G}{\partial S}\Rightarrow\dot{\lambda}_1=(\delta+\sigma)\lambda_1-a_4 \tag{27}$$

and

$$\dot{\lambda}_2 - r\lambda_2 = -\frac{\partial H_A}{\partial S} \Rightarrow \dot{\lambda}_2 = (r+\sigma)\lambda_2 + b_2(G+L), \tag{28}$$

wherein the usual transversality conditions apply. Differentiating (25) and (26) with respect to time and substituting into (27) and (28) yields the dynamic reaction functions of the Small country and the terrorist organization,

$$\dot{G} = (\delta+\sigma)(G-1) - \frac{\alpha a_4}{a_1} \tag{29}$$

and

$$\dot{A} = (r+\sigma)(A-1) - \frac{\beta b_2}{b_1}(G+L). \tag{30}$$

As expected, (29) and (30) reflect how the Small country's choice impacts that of the terrorist organization and vice-versa. It is important to stress that, differently from equations (10) and (11), these reaction functions now explicitly take into account the role of Large country security measures, L.

The Large country considers the dynamic reaction functions (29) and (30), as well as the state equation, in maximizing the objective function

$$\underset{L}{Max}\int_0^\infty [\xi(L-0.5L^2) - \varphi A + \rho_1 S + \rho_2 G]e^{-Rt}dt \tag{31}$$

subject to (22), (29) and (30). Note that the Large country's objective function takes into account both the security measures of the Small country and overall continental safety, reflecting full awareness.

The current-value Hamiltonian of the Large country's problem is,

$$H_L = [\xi(L - 0.5L^2) - \phi A + \rho_1 S + \rho_2 G]] + \lambda_L[\gamma L + \alpha G - \beta A - \sigma S - \Omega] + \mu_1\left[(\delta + \sigma)(G - 1) - \frac{\alpha a_4}{a_1}\right] + \mu_2\left[(r + \sigma)(A - 1) - \frac{\beta b_2(G + L)}{b_1}\right] \quad (32)$$

where λ and μ's are the Large country shadow prices of public safety and Small country and terrorists' actions. The first-order conditions are,

$$\frac{\partial H_L}{\partial L} = 0 \Rightarrow \xi(1 - L) + \lambda_L \gamma - \frac{\beta b_2}{b_1}\mu_2 = 0 \Rightarrow L = 1 + \xi^{-1}\left[\lambda_L \gamma - \frac{\beta b_2}{b_1}\mu_2\right], \quad (33)$$

$$\dot{\lambda}_L - R\lambda_L = -\frac{\partial H_L}{\partial S} \Rightarrow \dot{\lambda}_L = (R + \sigma)\lambda_L - \rho_1, \quad (34)$$

$$\dot{\mu}_1 - R\mu_1 = -\frac{\partial H_L}{\partial G} \Rightarrow \dot{\mu}_1 = R\mu_1 - \left\{\rho_2 + \alpha\lambda_L + \mu_1(\delta + \sigma) - \mu_2 \frac{\beta b_2}{b_1}\right\}, \quad (35)$$

and

$$\dot{\mu}_2 - R\mu_2 = -\frac{\partial H_L}{\partial A} \Rightarrow \dot{\mu}_2 = R\mu_2 - \{-\phi - \beta\lambda_L + \mu_2(r + \sigma)\}. \quad (36)$$

The steady state equilibrium value of the Small country's security and Large country's shadow price of continental public safety are easily found:

$$\dot{G} = 0 \Rightarrow G^* = 1 + \frac{\alpha a_4}{a_1(\delta + \sigma)}, \quad (37)$$

$$\dot{\lambda}_L = 0 \Rightarrow \lambda_L^* = \frac{\rho_1}{(R + \sigma)}, \quad (38)$$

The steady state equilibrium values of the remaining variables are found by solving the following equations:

$$\dot{A} = 0 \Rightarrow A = 1 + \frac{\beta b_2}{b_1(r + \sigma)}(G^* + L), \quad (39)$$

$$L = 1 + \xi^{-1}\left[\lambda_L^* \gamma - \frac{\beta b_2}{b_1}\mu_2\right], \tag{40}$$

and

$$\dot{\mu}_2 = 0 \Rightarrow \mu_2 = \frac{\left[\phi + \lambda_L^* \beta\right]}{[r + \sigma - R]}. \tag{41}$$

Substituting (38) into (40) yields the steady state value for the Large country's shadow value of terrorist attacks:

$$\mu_2^* = \frac{\left[\phi(R+\sigma) + \rho_1 \beta\right]}{[r + \sigma - R](R+\sigma)} \;. \tag{42}$$

Substituting (38) and (42) into (40) yields the steady state value of the Large country's security:

$$L^* = 1 + \xi^{-1}\left[\frac{\gamma\rho_1}{(R+\sigma)} - \frac{\beta b_2}{b_1}\frac{\left[\phi(R+\sigma) + \rho_1\beta\right]}{[r+\sigma-R](R+\sigma)}\right]. \tag{43}$$

Substituting (37) and (43) into (39) yields the steady state value of terrorist attacks:

$$A^* = 1 + \frac{\beta b_2}{b_1(r+\sigma)}(G^* + L^*). \tag{44}$$

Imposing $\dot{S} = 0$ in equation (22) and substituting (37), (43), and (44) yields the steady state value of continental public safety:

$$S^* = \sigma^{-1}\left[\gamma L^* + \alpha G^* - \beta A^* - \Psi\right]. \tag{45}$$

Because we have different equilibriums between the myopic and full awareness scenarios, it is appropriate to conduct a stability analysis to determine which one is stable and, therefore, more likely to persist in the long term.

4. Stability Analysis

The canonical system of both the myopic and full awareness model is:

$$\dot{S} = \gamma L + \alpha G - \beta A - \sigma S - \Psi \,, \tag{46}$$

$$\dot{G} = (\delta + \sigma)(G - 1) - \frac{\alpha a_4}{a_1} \,, \tag{47}$$

$$\dot{A} = (r + \sigma)(A - 1) - \frac{\beta}{b_1}(b_2 G + b_3 L) \,. \tag{48}$$

$$\dot{\lambda}_L = (R + \sigma)\lambda_L - \rho_1 \,, \tag{49}$$

$$\dot{\mu}_1 = R\mu_1 - \left\{ \rho_2 + \alpha\lambda_L + \mu_1(\delta + \sigma) - \mu_2 \frac{\beta b_2}{b_1} \right\}, \tag{50}$$

$$\dot{\mu}_2 = R\mu_2 - \left\{ -\phi - \beta\lambda_L + \mu_2(r + \sigma) \right\}. \tag{51}$$

We get the "Myopia Scenario" by setting $a_3 = a_4 = \rho_1 = \rho_2 = b_3 = 0$. In the full awareness scenario we have $b_2 = b_3 > 0$.

Substituting $L = 1 + \xi^{-1}\left[\lambda_L \gamma - \frac{\beta b_2}{b_1}\mu_2 \right]$ in the canonical system, we derive the Jacobian calculated at the steady state:

$$J = \begin{pmatrix} -\sigma & \alpha & -\beta & \gamma^2/\xi & 0 & \frac{\gamma\beta b_2}{\xi b_1} \\ 0 & \delta+\sigma & 0 & 0 & 0 & 0 \\ 0 & \frac{-\beta b_2}{b_1} & r+\sigma & \frac{-\gamma\beta b_2}{\xi b_1} & 0 & \frac{(\beta b_2)^2}{\xi b_1^2} \\ 0 & 0 & 0 & R+\sigma & 0 & 0 \\ 0 & 0 & 0 & -\alpha & R-\delta-\sigma & \frac{\beta b_2}{b_1} \\ 0 & 0 & 0 & \beta & 0 & R-r-\sigma \end{pmatrix} \quad (52)$$

Three eigenvalues can be easily derived and are given by:

$$e_1 = \delta+\sigma;\ e_2 = R-\delta-\sigma;\ e_3 = R+\sigma \quad (53)$$

The remaining eigenvalues are eigenvalues of the 3×3–matrix (59) below

$$\begin{pmatrix} -\sigma & -\beta & \frac{\gamma\beta b_2}{\xi b_1} \\ 0 & r+\sigma & \frac{(\beta b_2)^2}{\xi b_1^2} \\ 0 & 0 & R-r-\sigma \end{pmatrix} \quad (54)$$

All eigenvalues are real and are given by

$$\text{Eigenvalues} = \begin{pmatrix} \delta+\sigma \\ r+\sigma \\ R+\sigma \\ -\sigma \\ R-\delta-\sigma \\ R-r-\sigma \end{pmatrix} \quad (55)$$

Result: For both the case of myopia and full awareness, a saddle point stable equilibrium exists only if $R < \min(r+\sigma, \delta+\sigma)$.

The next section compares the myopic model with the full awareness model.

5. Scenario Comparison: Myopia vs. Awareness

In what follows we compare the different scenarios that emerge from the model. It is easy to see that the full awareness model yields a greater value of security for the Small country:

$$G^* = 1 + \frac{\alpha a_4}{a_1(\delta+\sigma)} > 1 = \overline{G}, \tag{56}$$

In regards to the Large country there are two possibilities: (1) the Large country is "extremely impatient," defined as $r+\sigma < R$, in which case the equilibrium is not stable and (2) the Large country is "relatively patient," defined as $r+\sigma > R$, in which case the equilibrium is stable (note, a sufficient condition for this outcome is that the Large country is more patient than the Small country, R<r).

In case (1) of an extremely impatient Large country, the full awareness model yields a greater level of security investments for the Large country [denoted L_1^*] than the myopic model:

$$L_1^* = 1 + \xi^{-1}\left[\frac{\gamma\rho_1}{(R+\sigma)} - \frac{\beta b_2}{b_1}\frac{[\phi(R+\sigma)+\rho_1\beta]}{[r+\sigma-R](R+\sigma)}\right] > 1 = \overline{L}. \tag{57}$$

In case (2) of a relatively patient Large country, the full awareness model may yield a smaller level of security investments for the Large country [denoted L_2^*] than the myopic model as long as L_2^*>0 and

$$L_2^* = 1 + \xi^{-1}\left[\frac{\gamma\rho_1}{(R+\sigma)} - \frac{\beta b_2}{b_1}\frac{[\phi(R+\sigma)+\rho_1\beta]}{[r+\sigma-R](R+\sigma)}\right] < 1 = \bar{L} \Leftrightarrow \frac{\gamma\rho_1}{(R+\sigma)} < \frac{\beta b_2}{b_1}\frac{[\phi(R+\sigma)+\rho_1\beta]}{[r+\sigma-R](R+\sigma)}. \quad (58)$$

The above results are quite important for the magnitude of terrorist activities in both countries. We can see that the greatest level of terrorist activity will occur in the full awareness model with an extremely impatient Large country. Since the model is explosive, because the equilibrium is unstable with an impatient Large country, this is the worst-case scenario.

When the Large country is patient [case (2)], the level of terrorist activity is still greater than in the myopic model:

$$A_1^* = 1 + \frac{\beta b_2}{b_1(r+\sigma)}(G^* + L_1^*) > A_2^* = 1 + \frac{\beta b_2}{b_1(r+\sigma)}(G^* + L_2^*) > \bar{A} = 1 + \frac{\beta b_2}{b_1(r+\sigma)}. \quad (59)$$

Therefore, the ranking of continental public safety is:

$$S_1^* = \sigma^{-1}\left[\gamma L_1^* + \alpha G^* - \beta A_1^* - \Psi\right] < S_2^* = \sigma^{-1}\left[\gamma L_2^* + \alpha G^* - \beta A_2^* - \Psi\right] < \bar{S} = \sigma^{-1}\left[\gamma\bar{L} + \alpha - \beta\bar{A} - \Psi\right],$$

(60)

If $\frac{\beta^2 b_2}{b_1(r+\sigma)} > \gamma$ it is easy to see that $s_1* < s_2*$. However, to obtain $s_2* < \overline{s}$ it is necessary to assume that the difference $\frac{\alpha^2 a_4}{a_1(\delta+\sigma)}$ is small enough so as to guarantee the inequality stands.

The above ranking results show that the full awareness model yields larger investments in security and more terrorist attacks than the myopic model. The scenario of an impatient Large country is the one with more investments in safety and, also, more terrorist attacks. However, it is unstable and explosive. Surprisingly, continental safety is higher in the myopic model than in the full awareness model.

6. Concluding Remarks

This paper analyzed a hierarchical differential game, with a Large and a Small country and a terrorist organization, as a model of the actual European refugee situation. Terrorists may enter Europe as refugees, taking advantage of the Open Doors Policy, to attack both countries.

Two scenarios were considered: myopia and full awareness. Countries are myopic when they ignore each other's security efforts, and the terrorist group only considers the weakest link's security effort. In the full awareness model, countries know that their expenditure on safety have a positive externality on the safety of their neighbors and on the whole continent.

A comparison between the scenarios shows that for an extremely impatient Large country, full awareness yields a greater level of security effort for the Large country, a greater level of security effort for the Small country, and, because of backlash, more terrorist attacks. One predicable result is that both countries will blame each other for their failures in curbing terrorist activities. Stability analysis, however, shows that this equilibrium is unstable.

The full awareness model with a patient Large country is stable and lies in between the full awareness model with an impatient Large country and the myopic model. Although it yields larger investments in security, this comes at a price of more terrorist attacks than the myopic model. As a consequence, continental safety is higher in the myopic model.

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