

A New Industry Concentration Index

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November 2, 2015

Abstract

We propose and analyze a new concentration index alternative to the Herfindahl-Hirschman Index (HHI). This new index emphasizes the concept of competitive balance. It is designed to preserve the convexity property of the HHI when a merger involves one of the m largest firms, but to decrease and thus to indicate an increase in competition when a merger is purely among the $(n - m)$ smallest firms.

Keywords: Horizontal mergers, dominant firms, small firms, competitive balance, concentration index.

JEL Classification Number: L10

1 Introduction

Merger analysis often involves a comparison between the pre- and post-merger degrees of concentration in a market. This degree of concentration matters since a high concentration measure is supposed to proxy for lack of competitiveness in that market. The standard index that is used to measure the level of concentration in an industry is the Herfindahl-Hirschman Index (*HHI*). The *HHI* possesses the so-called convexity property in that it increases whenever there is a 'mean preserving spread' of firms' market shares in an industry. Consequently, it yields a higher concentration level in response to *any* merger between firms.

HHI analysis is not always definitive however. In most jurisdictions, mergers that are suggested against by *HHI* are still allowed if they lead to significant cost savings that will increase welfare (for example 'authorization' in Australia or the 'rule of reason' in the United States). Hence, in reality, an increase in the *HHI* would not stop a merger if the merging parties could prove their cost savings. Unfortunately, some mergers that achieve welfare improvements through increased competition cannot overcome the judgment of the *HHI* in the absence of cost savings.

For example, suppose that there are three firms with percentage market shares of 70, 25, and 5 in Industry 1. *Ceteris paribus*, the *HHI* would deem that this industry is more concentrated and thus less competitive than Industry 2, which has market shares of 70, 15, and 15. But this is far from obvious since the dominant firm in Industry 2 faces two relatively small and potentially insignificant rivals who may simply 'follow' the lead of the dominant firm; whereas, in Industry 1, the relatively stronger firm with 25% market share may be able to provide greater competition to the dominant firm than can the two equally-sized, but smaller rivals of Industry.

Such a merger of two 15% market-share firms would raise the *HHI* and may be deemed anti-competitive even if the merger makes the newly created firm more competitive against an existing rival, creating positive welfare effects. Specifically, this merger raises the *HHI* from 5,350 pre-merger (using the standard convention of normalizing the *HHI* to be out of 10, 000) to 5,800

post-merger. Consequently, the merger would fall outside any safe harbours established in merger guidelines issued by competition regulators in Australia, the European Union, and the United States.¹

Noting these issues with the *HHI* (and other indices below), we propose and analyze an alternative index that emphasizes the concept of 'competitive balance'.² This new index is designed to have the convexity property of the *HHI* when a merger involves one of the m largest firms, but to decrease and thus to indicate an increase in competition when a merger is purely among the $(n - m)$ smallest firms.³ This allows for more direct use of the index evaluation that does not rely on the supplementary judgments discussed above (e.g., rule of reason, safe harbor), leaving administrators primarily to assess market structure and determine the appropriate number of firms (m) that dominate any given market.

2 A Brief Review of Concentration Indices

Let the market shares of n firms be listed as $v_1 \geq v_2 \geq \dots \geq v_n > 0$ where $\sum v_i = 1$. As mentioned above, the standard, most-prominent industry concentration index is the (*HHI*): $HHI(v_1, \dots, v_n) = (a_1 v_1 + \dots + a_n v_n)$ where $a_i = v_i$ so that the weights, a_1, \dots, a_n sum to one.

Another notable concentration index, the four-firm concentration ratio (*C4*), does not depend on the market shares of firms that are not the largest four firms: $C4(v_1, v_2, v_3, v_4) = (v_1 + v_2 + v_3 + v_4)$. Neither does it assign different weights to different market shares of the firms.

¹ Safe harbours define tolerable post-merger market concentration and/or concentration-change thresholds, above which proposed mergers are deemed likely to be anti-competitive. They are typically set via the *HHI* and changes in the *HHI*, i.e. ΔHHI .

Australia: $HHI < 2000$; or $HHI > 2000$ and $\Delta HHI < 100$:

The European Union: $HHI < 1000$; or HHI is between 1000 and 2000 and $\Delta HHI < 250$; or $HHI > 2000$ and $\Delta HHI < 150$.

The United States current: $HHI < 1000$; or HHI is between 1000 and 1800 and $\Delta HHI < 100$; or $HHI > 1800$ and $\Delta HHI < 50$:

The United States proposed: $HHI < 1500$; or $\Delta HHI < 100$.

See Yang and Pickford (2011).

² See Gans (2000) who provides a detailed evaluation of the 'competitive balance' concept.

³ Gugler et al. (2003) provide strong evidence that among mergers that increase profits, those involving larger firms achieve these profits by increasing their market power, while mergers involving smaller firms achieve higher profits by increasing efficiency.

There are a few other notable concentration indices. One, proposed by Hall and Tideman (1967), stresses the need to include the number of the firms in the calculation when measuring the concentration level of an industry (the number of firms measures the ease of entry into that particular industry). The Hall-Tideman concentration index (*HTI*) is

$$\text{HTI} \text{ is } \frac{1}{2 \left(\sum_{i=1}^n v_i \right) - 1}.$$

Hart (1967, p. 78) discussed an index of entropy, $E = - \sum_{i=1}^n v_i \log v_i$. Unlike the other indices presented thus far, it does not have a range of 0 to 1. Rather, it takes the value 0 when the market structure is a monopoly and takes a value far exceeding 1 when the market structure is perfect competition.

Finally, Dansby and Willig (1979) introduced alternative performance indices that measure the potential social gains from appropriate government interventions (such as anti-trust, regulatory, and de-regulatory actions). Their performance indices establish a welfare theoretic basis for indices such as *C4*, *HHI*, and others. Essentially, Dansby-Willig versions of these indices incorporate a weight that is the inverse of the price elasticity of the industry demand. Alternatively, Blackorby, Donaldson and Weymark (1982) (who use a Cobb-Douglas functional form) provided an index that assigns weights to not only firms' market shares but also to total output.

Central to our paper is that any merger would increase the measure of industry concentration according to all of the indices above, except for *C4* in which any merger beyond the largest four firms would have neither a negative nor positive effect on the measure of concentration unless the newly merged firm itself becomes one of the largest four firms. The proposed index below does not suffer from such rigidity.

3 CB* - The Competitive Balance Index

The 'competitive balance' index proposed here has different implications than the indices discussed above when horizontal mergers do not include the largest firm(s). Denote this index when there are m dominant firms in an industry as $CB^*(m)$, where $1 \leq m \leq n$. When $m = 1$, Firm i 's market share relative to that of the sole dominant firm is v_i / v_1 . It follows that the total market shares, relative to the largest firm's market share is $v_i / v_1 + v_2 / v_1 + \dots + v_n / v_1$.

We first consider this index when the market shares of the firms are measured in terms of only the largest firm's market share, $CB^*(1)$.

$$\begin{aligned}
 CB^*(1) &= \frac{1}{(v_i / v_1)^2 + (v_2 / v_1)^2 + \dots + (v_n / v_1)^2} \\
 &= \frac{1}{\left((v_i)^2 + (v_2)^2 + \dots + (v_n)^2 \right) \frac{1}{(v_1)^2}} \\
 &= \frac{(v_1)^2}{(v_i)^2 + (v_2)^2 + \dots + (v_n)^2}
 \end{aligned}$$

Observe that $CB^*(1) = \frac{(v_1)^2}{HHI}$.

Table 1 provides a few examples to illustrate the stark differences between the HHI and $CB^*(1)$.

Table 1: A Comparison of the HHI and $CB^*(1)$ under Different Market

Share Profiles

v_1	v_2	v_3	v_4	v_5	v_6	v_7	<i>HHI</i>	<i>CB*(1)</i>
70	5	5	5	5	5	5	.50	.97
70	30						.58	.84
51	26	11	11	1			.35	.74
51	37	11	1				.41	.64
40	37	11	11	1			.32	.5
40	37	9	9	5			.32	.51
33	33	33					.33	.33
50	50						.5	.5
55	45						.52	.6
55	35	10					.44	.7
55	25	20					.41	.75
40	40	20					.36	.44
40	40	10	10				.34	.47
40	20	20	20				.28	.57

Although there may be industries in which increasing the market share of the second largest firm could cause a reduction in the industry price, in many industries a reduction in price could not be achieved until a higher critical number of large firms is reached. For example, Lamm (1981, p. 75) reports empirical findings from the food retailing industry that in many urban markets “growth in the 3 largest firms’ shares have a significant positive effects on prices... In contrast, an increase in the market share of the fourth largest firm causes a reduction in food prices.” This clearly indicates that the number of dominant firms in a market may be greater than one which is critically important to the analysis of a potential merger. Thus, we now explore our index with $m > 1$ dominant firms, $CB^*(m)$.

$$\begin{aligned}
 CB^*(m) &= \frac{1}{\frac{v_1^2}{v_1^2 + \dots + v_m^2} + \frac{v_2^2}{v_1^2 + \dots + v_m^2} + \dots + \frac{v_n^2}{v_1^2 + \dots + v_m^2}} \\
 &= \frac{1}{\left[v_1^2 + v_2^2 \dots + v_n^2 \right] \frac{1}{v_1^2 + \dots + v_m^2}} \\
 &= \frac{v_1^2 + v_2^2 \dots + v_m^2}{v_1^2 + v_2^2 \dots + v_n^2}
 \end{aligned}$$

Observe that when $m > 1$, $CB^*(m) = \frac{v_1^2 + \dots + v_m^2}{HHI}$.

The following proposition describes how $CB^*(m)$ behaves in mergers that do and do not involve the largest firm.

Proposition 1

(1) If a merger does not involve the m largest firms and does not make the new firm one of the m largest firms, then $CB^*(m)$ decreases.

(2) If a merger involves one or two of the m largest firms, then $CB^*(m)$ increases.

Proof:

(1) Since $[v_1^2 + v_2^2 \dots + v_n^2]$ increases in any merger, $CB^*(m) = \frac{v_1^2 + \dots + v_m^2}{v_1^2 + v_2^2 + \dots + v_n^2}$

must decrease in any merger that does not involve any of v_1, v_2, \dots, v_m .

(2) Consider $CB^*(m) = \frac{v_1^2 + \dots + v_m^2}{v_1^2 + \dots + v_m^2 + \dots + v_n^2}$. Suppose Firm i and Firm j

merge such that $i \notin m$ and $j > m$. Thus, after the merger

$$CB^{*'}(m) = \frac{v_1^2 + \dots + v_{i-1}^2 + v_{i+1}^2 + \dots + v_m^2 + (v_i + v_j)^2}{v_1^2 + \dots + v_{i-1}^2 + v_{i+1}^2 + \dots + v_m^2 + (v_i + v_j)^2 + \dots + v_{j-1}^2 + v_{j+1}^2 + \dots + v_n^2}. \text{ Let}$$

$A = v_1^2 + \dots + v_m^2$ and $B = v_1^2 + \dots + v_m^2 + \dots + v_n^2$; hence $B > A$. Thus, $CB^*(m)$

becomes A/B and $CB^{*'}(m)$ becomes $\frac{A + (v_i + v_j)^2 - v_i^2}{B + (v_i + v_j)^2 - v_i^2 - v_j^2}$. Then

$$\begin{array}{ccc} > & & > \\ CB^{*'}(m) & = & CB^*(m) \text{ reduces to } Bv_j^2 + 2Bv_iv_j & = & 2Av_iv_j. \text{ Since } B > A, \text{ we} \\ < & & < \end{array}$$

obtain $CB^{*'}(m) > CB^*(m)$.

For the second case where Firm i and Firm j merge such that $i, j \notin m$, slightly modify the above argument. ■

The next proposition describes how much $CB^*(m)$ increases when Firm j merges with Firm i instead of with Firm i' where $v_i > v_{i'} > v_m > v_j$.

Proposition 2 Consider a merger M between Firm i and Firm j , and a merger M' between Firm i' and Firm j' , where $v_i > v_{i'} > v_m > v_j$. Then $CB^*(m, M) > CB^*(m, M')$.

Proof:

$$CB^*(m, M) = \frac{v_1^2 + \dots + v_{i-1}^2 + v_{i+1}^2 + \dots + v_m^2 + (v_i + v_j)^2}{v_1^2 + \dots + v_{i-1}^2 + v_{i+1}^2 + \dots + v_m^2 + (v_i + v_j)^2 + \dots + v_{j-1}^2 + v_{j+1}^2 + \dots + v_n^2}$$

and

$$CB^*(m, M') = \frac{v_1^2 + \dots + v_{i'-1}^2 + v_{i'+1}^2 + \dots + v_m^2 + (v_{i'} + v_{j'})^2}{v_1^2 + \dots + v_{i'-1}^2 + v_{i'+1}^2 + \dots + v_m^2 + (v_{i'} + v_{j'})^2 + \dots + v_{j'-1}^2 + v_{j'+1}^2 + \dots + v_n^2}$$

Let $A = v_1^2 + \dots + v_m^2$ and $B = v_1^2 + \dots + v_m^2 + \dots + v_n^2$. Thus, $CB^*(m, M)$

becomes $\frac{A + (v_i + v_j)^2 - v_i^2}{B + (v_i + v_j)^2 - v_i^2 - v_j^2}$ and $CB^*(m, M')$ becomes

$\frac{A + (v_{i'} + v_{j'})^2 - v_{i'}^2}{B + (v_{i'} + v_{j'})^2 - v_{i'}^2 - v_{j'}^2}$. Then $CB^*(m, M) > CB^*(m, M')$ reduces to

$\frac{A + v_j^2 + 2v_i v_j}{B + 2v_i v_j} > \frac{A + v_{j'}^2 + 2v_{i'} v_{j'}}{B + 2v_{i'} v_{j'}}$. Since $B > A + v_j^2$ and $2v_i v_j > 2v_{i'} v_{j'}$, we

obtain $CB^*(m, M) > CB^*(m, M')$. ■

The implication of the preceding proposition is that according to $CB^*(m)$, a merger between a small firm and a relatively large dominant firm will increase the concentration level in that industry more than will a merger between the same small firm and a relatively small dominant firm. Thus, Propositions 1 and 2 verify that $CB^*(m)$ satisfies the convexity property of the HHI when a merger involves one of the m largest firms, but decreases and thus indicates an increase in competition when a merger is purely among the $(n - m)$ smallest firms. Ultimately, the heterogeneity of different industries will inevitably require a case-by-case assessment of the correct value for m by the regulatory body holding jurisdiction over the

merger.

Table 2 considers several different market settings in an attempt to gauge how the *HHI* and *CB** respond to proposed mergers. In each row, merging firms' pre-merger market shares are denoted with a box around them. For instance, Rows 1 - 5 entail a situation in which there is one dominant firm and six identical smaller firms. All of the smaller firms merge in Row 1, five of the smaller firms merge in Row 2, and so on. The last two columns furnish the predicted changes in the two indices. Observe that *HHI* increases while *CB** decreases for each merger.

Table 2: Changes in *HHI* and *CB** Resulting from Mergers

v_1	v_2	v_3	v_4	v_5	v_6	v_7	<i>HHI</i>	<i>CB*</i> (1)	Δ in <i>HHI</i>	Δ in <i>CB*</i> (1)
70	5	5	5	5	5	5	.50	.97	.08	-.13
70	5	5	5	5	5	5	.50	.97	.06	-.09
70	5	5	5	5	5	5	.50	.97	.04	-.06
70	5	5	5	5	5	5	.50	.97	.02	-.03
70	5	5	5	5	5	5	.50	.97	.01	-.01
70	10	10	10				.52	.94	.06	-.10
70	10	10	10				.52	.94	.02	-.03
51	—	—	—				.35	.74	.15	-.22

		[25]	[12]	[12]				
51	[25]	[24]			.38	.68	.12	-.16
60	[20]	[10]	[10]		.42	.86	.10	-.17
60	20	[10]	[10]		.42	.86	.02	-.04
60	[20]	[20]			.44	.82	.08	-.13

4 Discussion: An Example

Heubeck et al. (2006) discuss the inadequacy of the *HHI* and other concentration indices that simply add pre-merger market shares of merging firms to approximate post-merger shares. The basic problem is that this method ignores second-order "industry wide strategic effects" that arise from post-merger competition where firms strategically alter quantity and/or pricing decisions.

Our example extends Heubeck's idea by considering that not only are "in-game" strategic actions affected by the merger, but that the very nature of competition may be affected as well. Specifically, we represent pre-merger competition using the standard dominant/fringe firms model. However, through the creation of competitive balance, post-merger competition transitions to Dixon's (1992) Edgeworthian price-quantity competition.

The general idea behind our index is that a merger between small firms that creates a more competitive mix of firms should be allowed, even if that mix yields a net reduction in the number of firms and an increase in the *HHI*. By changing the strategic nature of the game, economic welfare increases even though the *HHI* increases. Not surprisingly, this decreases the level of concentration according to *CB**.

The pre-merger status quo is such that the dominant firm sets its price based on residual demand, leaving the fringe firms to take that price and choose output accordingly. In the post-merger setup, however, the newly merged firm is on equal footing with the previously dominant firm and engages

in a price-quantity game where the remaining fringe firms take the price that results from the price-quantity competition as given.

The profit motive for the merging firms is akin to Caveat 3 (page 1245) of Levin (1990): they merge in order to eliminate redundancies in fixed costs. We adopt this motive for two reasons. First, the elimination of fixed-cost redundancies is sufficient to guarantee the profitability of the merger. Second, we wish to avoid variable production efficiencies as their presence would possibly lead to merger approval even if the *HHI* suggests otherwise (recall the discussion in the Introduction).

Our specific example is as follows.⁴ Demand in the market is $Q = 90 - P$, where Q is the total quantity produced by all firms and P is the market price. The dominant firm has a cost of $C = 50 + 0.5q_d^2$ where q_d is the quantity it produces. Each of the four smaller fringe firms has a cost of $C = 45 + q_f^2$ where q_f is the quantity each such firm produces. Inverse Residual Demand for the dominant firm's product is therefore $P = 30 - \frac{1}{3}q_r$. Equilibrium in this market has the dominant firm produce $q_d = q_r = 18$ units and set a price of \$24, generating a profit of \$220. Supposing the fringe firms follow the dominant firm by accepting that price, each then produces 12 units and earns profits of \$99.

Now let two of the fringe firms merge in an effort to eliminate fixed cost redundancies and become strategically more competitive against the dominant firm. Assume that the remaining two fringe firms remain on the fringe, taking the equilibrium price arising between the previously dominant firm and the newly merged firms as given. That equilibrium price ascribes to the Dixon (1992) Edgeworthian model equilibrium where the competitive price ($P = MC$) is achieved, given the post-merger residual demand curve.⁵

The marginal cost curves of the firms that merged yield the cost function of

⁴ Although this example is stylized and simple for illustrative purposes, the qualitative results will hold for many other parameter values.

⁵ The Edgeworthian price-quantity model is an extension of the Edgeworth-Bertrand model which itself is an extension of the standard Bertrand model. For those readers unfamiliar with the Edgeworthian model, our example is simplistic enough to think of that model as a standard Bertrand model with the caveat that the low-price dominant firm is not required to supply the entire market, but is given the choice of how many units to supply, leaving the residual demand to the higher-priced dominant firm.

this newly-created dominant firm, $C = 50 + 0.5q_d^2$, where the reduction in fixed costs has put the merged firm on equal footing with the previously uniquely dominant firm. Given that there are only two remaining fringe competitors, the Inverse Residual Demand curve facing the two Bertrand competitors is now $P = 45 - 0.5q_r$.

The equilibrium Edgeworthian price in the game between the dominant firms is easily verified to be \$22.50. At that price the previously-dominant firm as well as the newly-merged dominant firm each produces 22.5 units and each earns \$203.13 in profits. Each of the two remaining fringe firms now produces 12.25 units and each earns \$92.81 in profits. The reduction in fixed costs accompanying the merger leads to joint profits for the merged firm that are greater than the summed individual profits they would have earned by remaining on the fringe.

Post-merger, the market price is lower than the pre-merger price and marginal costs have not changed and thus the merger leads to an increase in welfare. Pre-merger, the market shares were (1/3, 1/6, 1/6, 1/6, 1/6). Post-merger, they become (1/3, 1/3, 1/6, 1/6), leading to pre- and post-merger HHI measures of 2222 and 2777 respectively. Alternatively, the pre- and post-merger $CB^*(1)$ measures are 0.0 and 0.4. Thus, this example illustrates how a merger that is welfare enhancing can decrease CB^* , but increase the HHI .

Extending the example further, we can use $CB^*(2)$ by then allowing the two remaining fringe firms to merge in order to compete with the two dominant firms, putting all three firms on equal footing. It can be easily confirmed that once the final two fringe firms merge, the Edgeworthian equilibrium is for all three firms to charge \$22.50, resulting in output by each firm of 22.5 units. There is no welfare loss since the pre- and post-merger prices are the same. Using the pre- and post-merger market shares of (1/3, 1/3, 1/6, 1/6) and (1/3, 1/3, 1/3) yield the respective pre- and post- HHI measures of 2777 and 3333. Likewise, the pre- and post-merger $CB^*(2)$ measures are 0.8 and 0.66666, once again illustrating how a merger that is welfare enhancing can decrease CB , but increase the HHI .

Finally, in our index the analyst must make a judgment about how many

firms to include in m . This choice is crucial since once m is chosen, a merger involving two small firms decreases the index, and a merger involving at least one of the large firms raises the index. One relevant question then is "what determines m ?" In some of these cases, industry analysts may have already determined it empirically. For example, Lamm (1981) reports that in the food retailing industry, "growth in the 3 largest firms' shares have a significant positive effects on prices... In contrast, an increase in the market share of the fourth largest firm causes a reduction in food prices"). In other cases, like the example we have just provided above, a natural gap between the market shares of firms may provide strong clues about m . E.g., if the market shares profile is $(1/3, 1/6, 1/6, 1/6, 1/6)$, then it would be straightforward to deduce that $m = 1$, whereas if that profile is $(1/3, 1/3, 1/6, 1/6)$, then one can deduce that $m = 2$. In the end, the simple fact is that market heterogeneity and complexities will require case-by-case assessments of m .

5 References

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